Math 1050 College Algebra
Rubric for Sample Final Exam 3

General Guidelines

- Only perfectly completed problems should receive full credit. If the solution contains an arithmetic error, at least one point must be deducted
- Only award and deduct an integer number of points

(6 points) 1. True or false.

(F) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 0 & 8 \end{bmatrix}$.

(F) If $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{2}{x+1}$, then $(f+g)(x) = \frac{3}{2x}$.

(T) Let $c_1 = -1$ and $c_n = 2c_{n-1} + 1$. Then $c_3 = -1$.

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For each, award 2 points for the correct response and 0 points for wrong or blank answers

Question 2.

(9 points) 2. Fill in the blank.

(a) The exact value of $3^{\log_3 17}$ is **(17)**

(b) The minimum value of $y = 2x^2 + 8x - 3$ is $y =$ **(-11)**

(c) The domain of the function $f(x) = \sqrt{x+4}$ is **($x \geq -4$)**

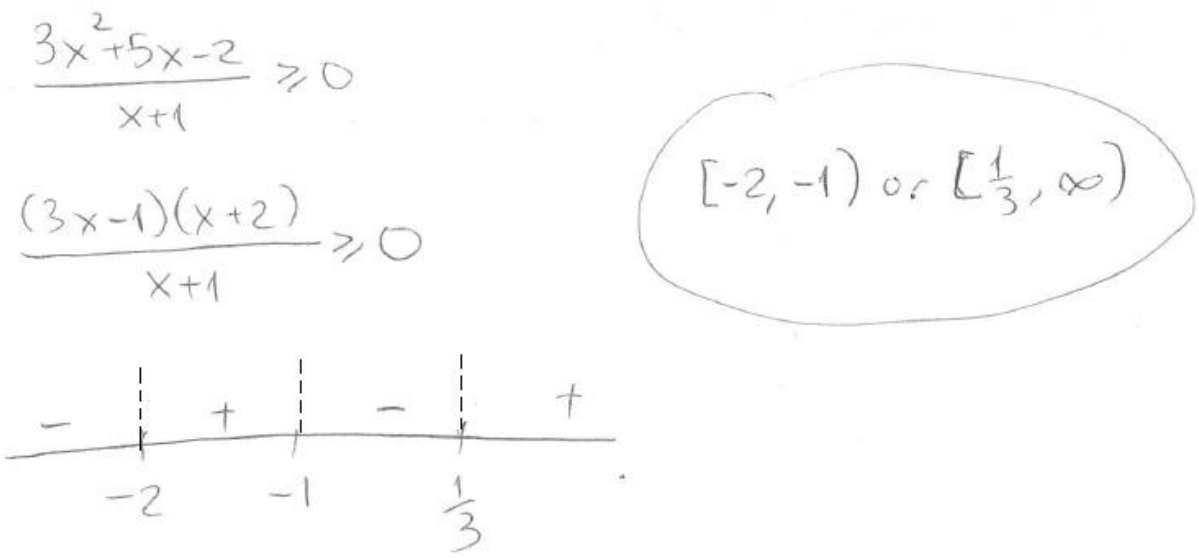
Award 3pt for each correct answer

(a) +3 for correct answer

(b) +3 for correct answer

(c) +3 for correct answer

(10 points) 3. Solve the inequality. Write your answer in interval notation. $\frac{3x^2+5x-2}{x+1} \geq 0$.



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+3 for correctly factoring the numerator
+1 for identifying critical points -2, -1, 1/3

- Do not need to be labeled as such to receive the points

+1 for recognizing the need to evaluate the behavior of the function in different regions
+2 for correctly implementing a method to evaluate the behavior of the function in different regions (using a number line, logical connectives, or graphing)

- Award +1 if a student makes a small error in work
  Student must show and communicate clearly to be awarded the partial point
- Award +0 if a student shows no ability to carry out the chosen method

+1 for correctly identifying the regions for which the function is greater than or equal to zero
+1 for correct end points (that is -2 and 1/3 are included in the solution and -1 is not included)
+1 for correct interval notation

- Award based on the answer from previous step whether or not that is the correct interval

Note:

- If a student begins by "cross multiplying", award only one point for correct interval notation
- If a student avoids checking regions, the student is only eligible for maximum of 4pt (1pt for correct interval notation, 1pt for correct end points and 2pt for correct calculations)

(15 points) 4. Consider the rational function $f(x) = \frac{6x}{(x+3)(x-2)}$.

(a) State its domain.

$D = \{x \mid x \neq -3, x \neq 2\}$

RUBRIC

(a) **Total of 2 points for part (a)**

+1 for $x \neq -3$
+1 for $x \neq 2$

- Domain may be given in set notation, interval notation, or just by identifying the restrictions.
- Award 1pt if other restrictions are also given

(b) Find all intercepts of its graph, if any.

$y_{int} = (0,0) = x_{int}$

RUBRIC

(b) **Total of 2 points for part (b)**

- Student may simply say (0,0) , then award 2pt
- If a student says only x=0, award 1pt
- If a student says only y=0, award 1pt
- If a student writes only 0, award 1pt
- Take away a point if another intercept is also given

(c) Find all asymptotes of its graph.

V.A: $x = -3, x = 2$ H.A: $y = 0$

RUBRIC

(c) **Total of 3 points for part (c)**

+1 for H.A.
+2 for V.A.

(d) Determine whether its graph crosses a non-vertical asymptote. **Justify your answer**.

$\frac{6x}{(x+3)(x-2)} = 0$ $\quad 6x = 0 \quad x = 0 \quad$ crosses

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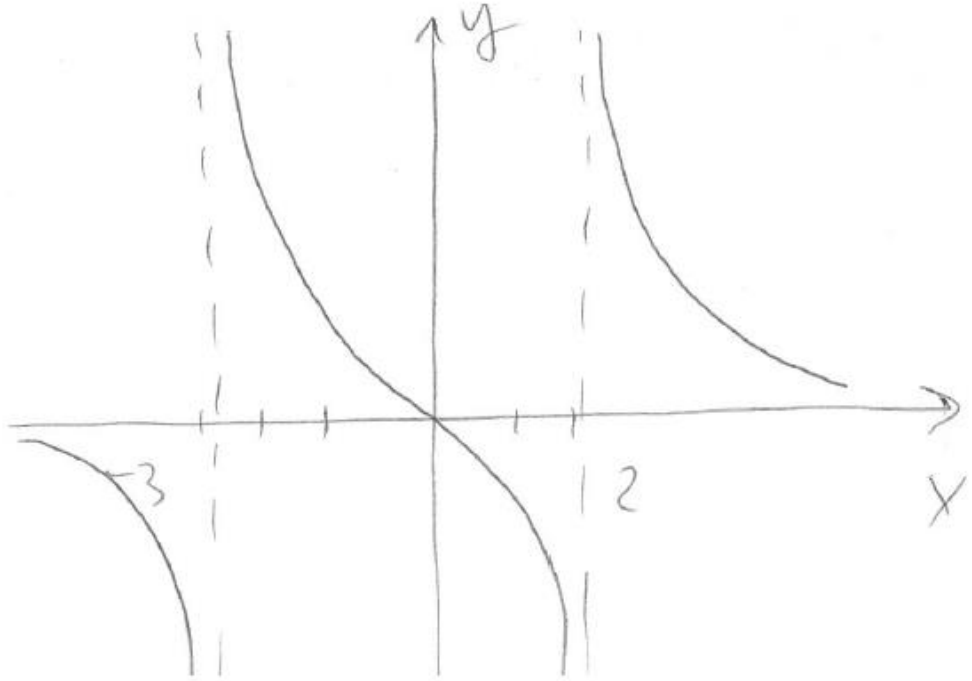
(d) **2 points total for part (d)**

+1 for $\frac{6x}{(x+3)(x+2)} = 0$

+ 1 for concluding that the graph does not crosses the horizontal asymptote

(e) Use the above information and other appropriate points to draw its graph.



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(e) **Total of 6 points for part (e)**

+1 for showing the x- and y - intercepts
+1 for plotting the asymptotes
+1 for correct behavior in Region 1
+1 for correct behavior in Region 2
+1 for correct behavior in Region 3
+1 for showing correctly asymptotic behavior

(10 points) 5. Find the partial fraction decomposition of the rational expression.

$$\frac{x-3}{(x+2)(x+1)^2}$$

$$\frac{x-3}{(x+2)(x+1)^2} = \frac{A}{x+2} + \frac{B}{(x+1} + \frac{C}{(x+1)^2} = \frac{-5}{x+2} + \frac{5}{x+1} + \frac{-4}{(x+1)^2}$$

$$x-3 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$$

$x=-2 \quad -5 = A$

$x=-1 \quad -4 = C$

$x=0 \quad -3 = A + 2B + 2C$

$-3 = -5 + 2B - 8$

$2B = 10$

$B = 5$

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+4 for specifying the correct form for the partial fraction: $A/(x + 2) + B/(x + 1) + C/(x + 1)^2$.
Note: the alternate form $A/(x + 2) + (Bx + C)/(x + 1)^2$ may also be used.

+2 for multiplying both sides by the denominator: $(x + 2)(x + 1)^2$.

+3 for correctly solving for the unknowns $A = -5$, $B = 5$ and $C = -4$.
Note: having used the alternate form, the correct values are $A = -5$, $B = 5$ and $C = 1$

+1 for displaying the correct answer
Note: for the alternate form, the correct answer is $-5/(x + 2) + (5x + 1)/(x + 1)^2$.

(10 points) 6. An arithmetic sequence has a common difference equal to 10 and its $6th$ term to 52.

(a) Find its $15th$ term.

$d = 10$

$a_6 = a_1 + 5d = a_1 + 50 = 52$

$a_1 = 2$

$a_{15} = a_1 + 14d = 2 + 14 \cdot 10 = 142$

(b) Use appropriate formula to find the sum of first 15 terms of this sequence.

$S_n = \frac{a_1 + a_n}{2} \cdot n$

$S_{15} = \frac{2 + 142}{2} \cdot 15 = 1,080$

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(a) Total of 6pt for part (a)

+3 for correctly finding the first term (1pt for correct set up and 2pt for correct arithmetic)
+1 for correct formula of the n-th term
+2 for correctly finding the 15th term

Note.

- If a student does not use the n-th term formula, but clearly communicates how he/she found the first term correctly and later the 15th term correctly, award full points

- If a student does not use the n-th term formula, but clearly communicates how he/she found the first term and later the 15th term, but the answer is not correct, award no more than 4pt for the whole question

(b) Total of 4pt for part (b)

+1 for correct formula of the sum
+2 for correct calculations
+1 for correct final answer

Note. If the 15^{th} term and/or the first term was calculated incorrectly in part (a), but in part (b), formula and calculations are correct according to the values form part (a), award up to 4pt for the whole question

(10 points) 7. Use matrices (row operations/Gauss elimination) to solve the following system of linear equations. **No points will be given if the system is solved by other methods.**

$$\begin{cases} x+y+z=7 \\ 2x-y+z=-4 \\ -x+2y-z=8 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 2 & -1 & 1 & -4 \\ -1 & 2 & -1 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -3 & -1 & -18 \\ 0 & 3 & 0 & 15 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 3 & 0 & 15 \\ 0 & -3 & -1 & -18 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & -3 & -1 & -18 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & -1 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x = -1$
$y = 5$
$z = 3$

RUBRIC

+1 for setting up $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 2 & -1 & 1 & -4 \\ -1 & 2 & -1 & 8 \end{array}\right]$ (all or nothing)

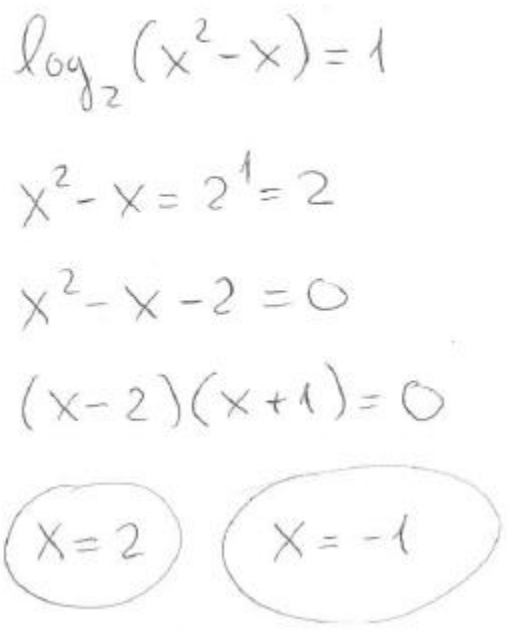
+4 for correct row operations
- Deduct 1 point for 1-2 small arithmetic errors with correct row operations
- Deduct 2 points for multiple arithmetic errors with correct row operations
- Only give points for correct row operations

+2 for arriving with the reduced echelon form

+2 for correctly setting up and manipulating the equations to solve for x, y, and z
- Student may choose to continue row operation until arriving with the identity matrix

+1 for correct answer as x =-1, y =5, z = 3

(10 points) 8. Solve the equation. $\log_2(x^2 - x) = 1$.



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+3 for $x^2 - x = 2$ (all or nothing)
+1 for arriving at $x^2 - x - 2 = 0$
+2 for factoring to $(x - 2)(x + 1) = 0$
+2 for arriving at $x = 2$, $x = -1$
+2 for concluding that both answers are valid solutions

Note. If a student begins with the equation like: $x^2 - x = 1$, the student can only receive 1pt for the whole question

(10 points) 9. Suppose John has \$4,000 to invest and wants \$5,000 to buy a hot tub. He finds a bank offering 5.5% interest compounded quarterly. How long will he have to leave his money in the account to have \$5,000? Round your answer to one decimal place.

$$5{,}000 = 4{,}000\left(1+\frac{0.055}{4}\right)^{4t}$$

$$\frac{5}{4} = (1.01375)^{4t}$$

$$\ln\frac{5}{4} = \ln 1.01375^{4t}$$

$$\ln\frac{5}{4} = 4t \cdot \ln 1.01375$$

$$t = \frac{\ln\frac{5}{4}}{4 \cdot \ln 1.01375} = 4.1$$

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+2 for $A = P(1+\frac{r}{n})^{nt}$ (all or nothing)

- award the points if correct formula is used to substitute into but not explicitly stated before substituting in values

+1 for correctly identifying A = 5,000, P = 4,000, r = 0.055, n=4
+1 for dividing both sides by the factor standing by P
+1 for recognizing the need to take ln of both sides
+4 for correct calculations
+1 for correct answer

- If a student only writes the final answer without showing the work, award 1pt

Note.

- If the wrong formula is used, student can earn maximum of 4 points (+1 for identifying A, P, m, and t; +1 for showing understanding of correct order of operations and +2 for correct arithmetic)

- If a student does not isolate for the term with the exponent before taking ln of both sides, but uses properties of logarithms correctly and arrives with the correct answer, award full credit

- If a student does not isolate for the term with the exponent before taking ln of both sides, and that results with the wrong answer, the student is only receive maximum of 4pt for the whole questions (The points distribution is a s follow: 2pt for the correct formula, 1pt for correctly identifying A, P, r, n, one point for some correct arithmetic and one point for demonstrating the ability to round an answer to one decimal place)

(10 points) 10. Find the sixth term of the expansion of $(1 - 2a^2)^7$ using the Binomial Theorem, and simplify it.

$$(k+1)\text{th} = \binom{n}{k}(1)^{n-k}(-2a^2)^k$$

$$= \binom{7}{5}(1)^{7-5}(-2a^2)^5$$

$$= \frac{7!}{5!2!}(-32)a^{10}$$

$$= \frac{7 \cdot 6}{2}(-32)a^{10}$$

$$= 21 \cdot (-32)a^{10} = -672a^{10}$$

RUBRIC

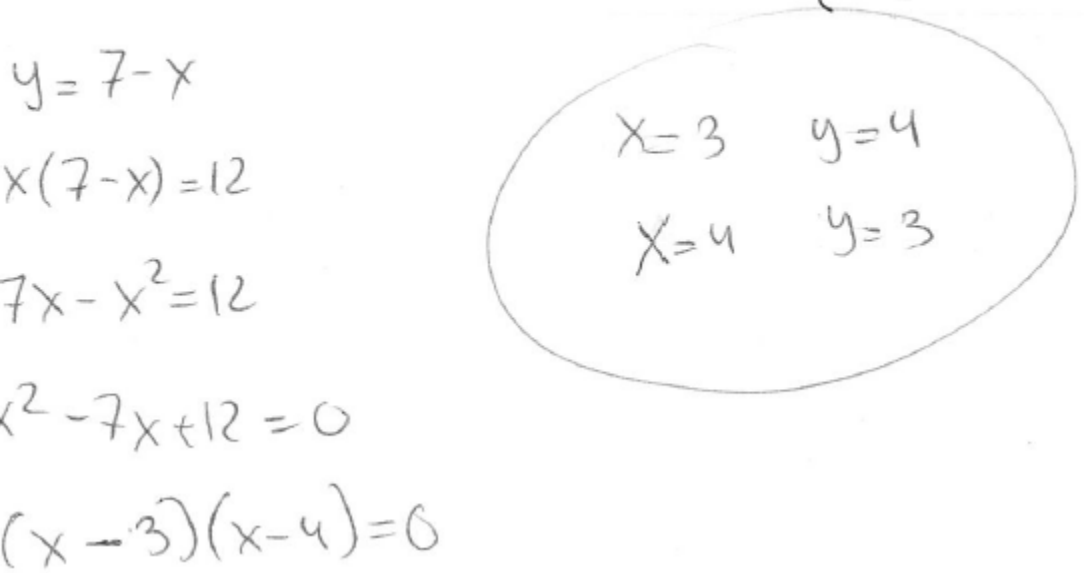
+4 for correctly setting up the sixth term
+6 for correctly calculating the term, that is

- +1 for correct calculation of C(7,5) = 21 (calculator is accepted, that is student does not need to show factorials
- +1 for correct exponents
- +2 for correctly performing the exponent in the term containing the variable
- +1 for correct treatment of the minus
- +1 for correct answer

Note.

- Student may choose to expand the binomial (using Binomial Thm or Pascal's Triangle) and identify the term from there, if that is done correctly award all points. If errors are encounter, award points as outlined above (for setting up, for the coefficients, for exponents, for performing the exponents correctly, for calculating the coefficients correctly)
- If a student expands the binomial up to the sixth term and notices that it is the term needed, award all points if the term and the expansion are correct
- Deduct a point if the needed term is calculated correctly and the answer is correct, but the whole expansion has one or two small arithmetic errors
- Deduct 2pt if multiple errors are encountered in the expansion, but the needed term is calculated correctly and the answer is correct
- If a student multiplies out (foil) the binomial and get the correct answer, award no more than 4pt for the whole question
- If a student multiplies out (foil) the binomial and does not get the correct answer, award no more than 2pt for the whole question

(10 points) 11. Solve the system of nonlinear equations: $\begin{cases} xy = 12 \\ x + y = 7 \end{cases}$



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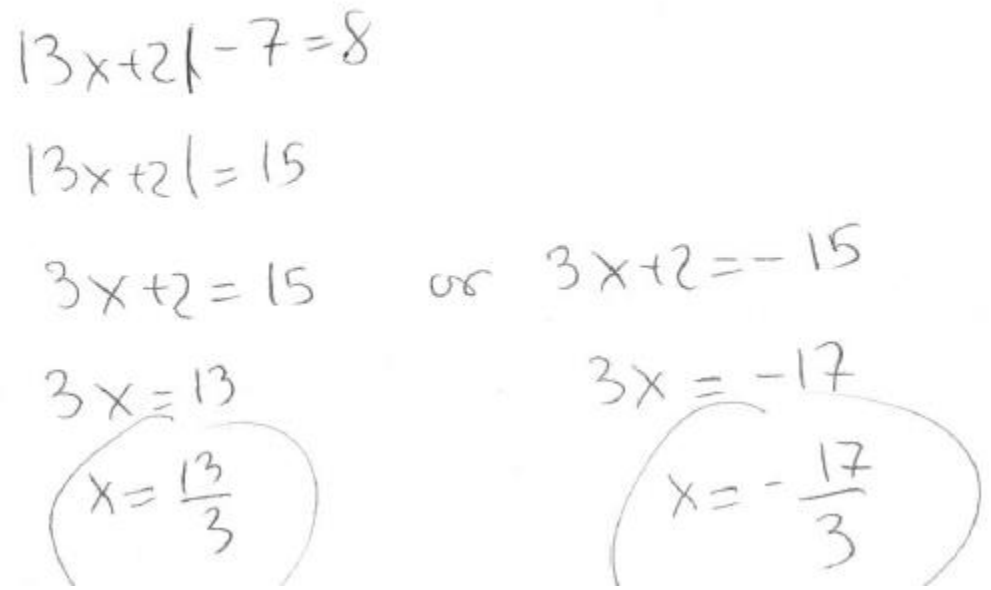
+1 for isolating one variable from the first equation
+1 for correctly substituting the one variable into the second equation
+2 for correct calculations and arriving at correct equation with one variable
+1 for correctly factoring the quadratic equation
+2 for correct values of the first variable
+2 for correctly solving for the second variable
+1 for correctly stating the answer, either as ordered pairs or clearly communicating the solution as pairs of x and y values

(10 points) 12. Solve equation. $|3x + 2| - 7 = 8$.



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+1 for adding 7 to both sides
+1 for arriving with $|3x + 2| = 15$
+4 for correctly transposing the absolute value statement into two equations

- Note: Students may choose to solve by squaring both sides of the inequality. If this method is done mathematically correct, the student should be awarded all the points in this section.
- Any wrong answer that shows inability to translate this absolute value equality will receive none of these 4 points

+1 for subtracting 2 from both sides

+1 for dividing both sides by 3

+2 for arriving with $x = \frac{13}{3}, \quad x = -\frac{17}{3}$

Note:

- For students that do not begin by isolating the absolute value on one side of the equation, the student is only eligible to receive maximum of 4pt for the whole question (that is, 2pt for arithmetic and up to 2 additional point if the grader feels the student demonstrations some understanding of transposing absolute value statements)

(10 points) 13. Your factory produces lemon-scented widgets. You know that each unit is cheaper, the more you produce. But you also know that costs will eventually go up if you make too many widgets, due to the costs of storage of the overstock. The guy in accounting says that your cost for producing x thousands of units a day can be approximated by the formula $C = 0.04x^2 - 8.504x + 25302$.

(a) Find the daily production level that will minimize your costs. **Round your answer to one decimal place.**

$$X = -\frac{b}{2a} = -\frac{-8.504}{2(0.04)} = 106.3$$

(b) What is the minimum daily production cost?

$$C_{min} = 0.04(106.3)^2 - 8.504(106.3) + 25302 = 24,850$$

RUBRIC

(a) **Total of 6 points for part (a)**

+1 for recognizing the need to find the vertex of the quadratic function
+2 for correct formula
+2 for correctly calculating the vertex
+1 for correct answer

(b) **Total of 4 points for part (b)**

+1 for correctly substituting in the C
+2 for correct calculations
+1 for correct answer

Note.

- If a student found the x value incorrectly in part (a), but the min cost is correct according to his/her vale of x, award 4pt for part (b)

(10 points) 14. Let $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{1}{x+1}$.

(a) Find the quotient $(\frac{f}{g})(x)$ and simplify it.

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{x}{x+1}}{\frac{1}{x+1}} = \frac{x}{x+1}\,\frac{x+1}{1} = x$$

(b) Find the domain of the function $(\frac{f}{g})$ and state the answer in the set notation.

$$D\left(\frac{f}{g}\right) = \{x \mid x \neq -1\}$$

RUBRIC

(a) **Total of 7 points for part (a)**

+2 for correctly substituting function f and g into the quotient
+4 for correct calculations
+1 for correct answer

(b) **Total of 3 points for part (b)**

+1 for the restriction $x \neq -1$
+1 for correct final answer
+1 for correct set notation

(10 points) 15. Let $f(x) = 3^{x+1} - 1$.

(a) Determine the domain

$(-\infty, \infty)$

(b) Find all intercepts of its graph

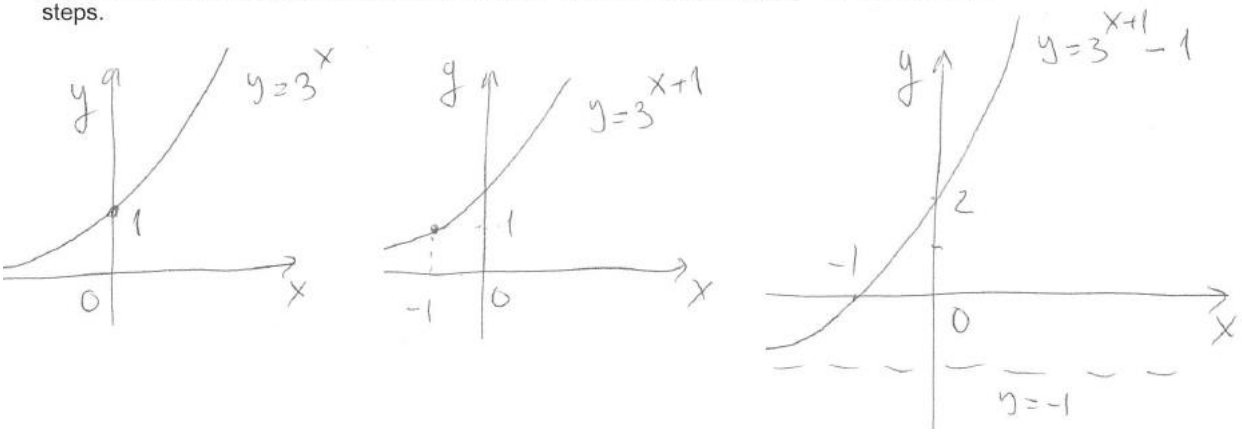
$y_{int} = (0, 2)$ x_{int}: $3^{x+1} - 1 = 0$ $3^{x+1} = 1 = 3^0$ $x+1 = 0$

$x_{int} = (-1, 0)$ $x = -1$

(c) Find all aymptotes of its graph

H.A: $y = -1$

(d) Graph the function $f(x)$ using transformations. Start with graphing $g(x) = 3^x$ and show all steps.



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(a) **Total of 2 points for part (a)**

+2 for correct domain (stated in any form)

(b) **Total of 3 points for part (b)**

+1 for correct y-int

+2 for correctly calculating the x-int. (1pt for correct setting and 1pt for correct calculations leading to the correct answer)

(c) **Total of 2 points for part (c)**

+2 for correctly stating the horizontal asymptote

- If a student simply says: y=-1, award all points for part (c)
- If a student just says: -1 (without specifying if the asymptote is vertical or horizontal), award 1pt
- If a student writes vertical asymptote at 1 (or V.A. x = -1), award 1pt

(d) **Total of 3 points for part (d)**

+1 for maintaining the shape of the exponential function

+1 for horizontal shift 1 unit to the left

+1 for vertical shift 1 unit down