# Math 1050 College Algebra
## Rubric for Sample Final Exam 2

General Guidelines
- Only perfectly completed problems should receive full credit. If the solution contains an arithmetic error, at least one point must be deducted
- Only award and deduct an integer number of points

(6 points)  1. True or false.

( F )  $\dfrac{\log_3 12}{\log_3 6} = \log_3 2.$

( F )  $f(x) = 0$ is one-to-one function.

( T )  The domain of the function $f(x) = \sqrt{2-x}$ is $(-\infty, 2]$.

**RUBRIC**

For each, award 2 points for the correct response and 0 points for wrong or blank answers

**Question 2.**

(9 points)  2. Fill in the blank.

(a) The common difference of the arithmetic sequence $2, 5, 8, \ldots$ is  $3$

(b) The graph of the function $f(x) = 3x^5 + 4x^2 + x$ may have at most  $4$  turning points.

(c) The remainder of the division $\dfrac{2x^2 - x + 5}{x+1}$ is  $8$

**RUBRIC**

**Award 3pt for each** correct answer

(a) +3 for the correct answer

(b) +3 for the correct answer

(c) +3 for the correct  answer
- Award 1pt  for answer like  $\dfrac{8}{x+1}$

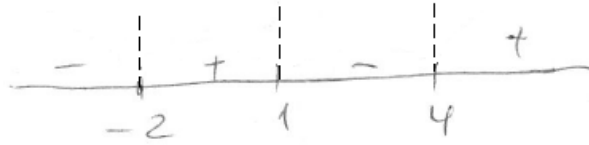(10 points) 3. Solve the inequality. Write your answer in interval notation. $\frac{2}{x+2} < \frac{1}{x-1}$.

$$\frac{2}{x+2} < \frac{1}{x-1}$$

$$\frac{2}{x+2} - \frac{1}{x-1} < 0$$

$$\frac{2(x-1)-(x+2)}{(x+2)(x-1)} < 0$$

$$\frac{2x-2-x-2}{(x+2)(x-1)} < 0$$

$$\frac{x-4}{(x+2)(x-1)} < 0$$

$$-\quad + \quad - \quad +$$
$$-2 \qquad 1 \qquad 4$$

$$(-\infty, -2) \text{ or } (1,4)$$

**RUBRIC**

+1 for subtracting the fraction from both sides to create zero on one side

+1 for arriving at $\frac{2(x-1)-(x+2)}{(x+2)(x-1)} < 0$, or something equivalent that shows correct common denominator

+1 for correctly distributing minus and arriving at $x - 4$ in the numerator

+1 for identifying critical points  -2, 1 and 4
- Do not need to be labeled as such to receive the points

+1 for recognizing the need to evaluate the behavior of the function in different regions

+3 for correctly implementing the method to evaluate the behavior of the function in different regions (using a number line, logical connectives, or graphing )
- Award +2 if a student makes a small error in work
- Award +1 if a student makes two or more errors
  Student must show and communicate clearly to be awarded the partial point
- Award +0 if a student shows no ability to carry out the chosen method

+1 for correctly identifying the regions for which the function is less than zero

+1 for correct interval notation
- Award based on the answer from previous step whether or not that is the correct interval

Note:
- If a student begins by "cross multiplying", award only one point for correct interval notation
- If a student avoids checking regions, the student is only eligible for maximum of 4pt (1pt for correct interval notation, and 3pt for correct calculations leading to the correct critical points)

(15 points) 4. Consider the rational function $f(x) = \frac{x^2}{x^2-1}$.

(a) State its domain.

$$x^2-1=0 \qquad x=\pm 1 \qquad D=\{x \mid x \neq \pm 1\}$$

**RUBRIC**

(a) **Total of 2 points for part (a)**

+1 for $x \neq -1$

+1 for $x \neq 1$

- Domain may be given in set notation, interval notation, or just by identifying the restrictions.
- Award 1pt if other restrictions are also given

(b) Find all intercepts of its graph, if any.

$$x_{int} = y_{int} = (0,0)$$

**RUBRIC**

(b) **Total of 2 points for part (b)**

- Student may simply say (0,0) , then award 2pt
- If a student says only x=0, award 1pt
- If a student says only y=0, award 1pt
- If a student writes only 0, award 1pt
- Take away a point if another intercept is also given

(c) Find all asymptotes of its graph.

$$V.A. \quad x=\pm 1, \qquad H.A: y=1$$

**RUBRIC**

(c) **Total of 3 points for part (c)**

+1 for H.A.

+2 for V.A.

(d) Determine whether its graph crosses a non-vertical asymptote. **Justify your answer.**

$$\frac{x^2-1}{x^2} = 1 \qquad x^2-1 = x^2$$
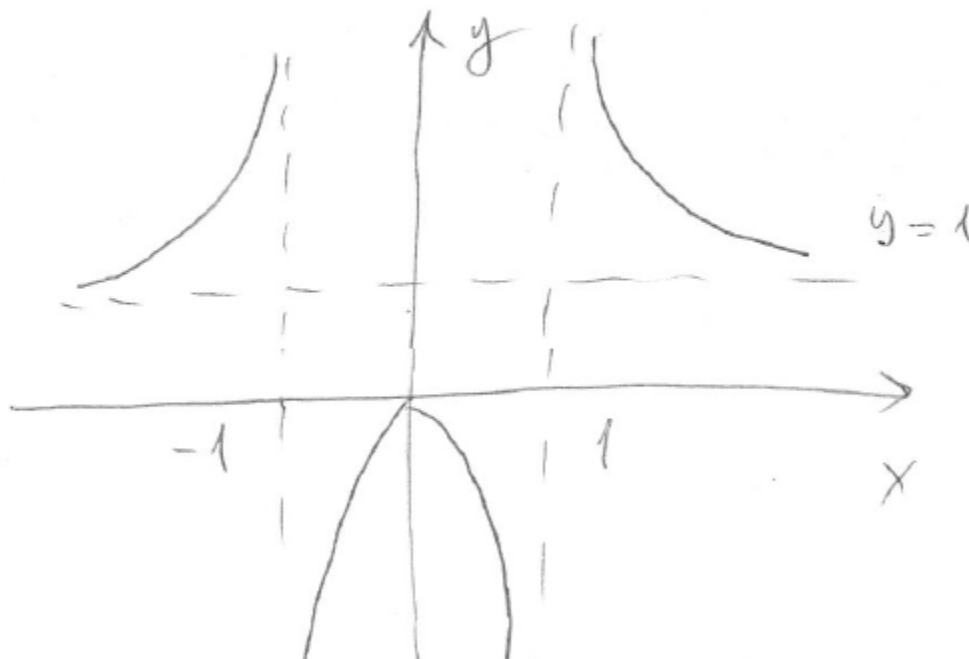$$-1 \neq 0$$

Does not cross

**RUBRIC**

(d) **2 points total for part (d)**

+1 for $\frac{x^2-1}{x^2} = 1$

+ 1 for concluding that the graph does not crosses the asymptote

(e) Use the above information and other appropriate points to draw its graph.



**RUBRIC**

(e) **Total of 6 points for part (e)**

+1 for showing the x-intercepts and y-intercepts

+1 for plotting the asymptotes

+1 for correct behavior in Region 1

+1 for correct behavior in Region 2

+1 for correct behavior in Region 3

+1 for showing correctly asymptotic behavior

(10 points) 5. Find the partial fraction decomposition of the rational expression.

$$\frac{1}{(x+1)(x^2+4)}$$

$$\frac{1}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} = \frac{1/5}{x+1} + \frac{-\frac{1}{5}x+\frac{1}{5}}{x^2+4}$$

$$1 = A(x^2+4) + (x+1)(Bx+C)$$

$x=-1$  $1 = A \cdot 5$  $\boxed{A = \frac{1}{5}}$   $x=1$   $1 = 5A + 2(B+C)$

$x=0$  $1 = 4A + C$   $1 = 1 + 2B + \frac{2}{5}$

$1 = \frac{4}{5} + C$   $2B = -\frac{2}{5}$

$\boxed{C = \frac{1}{5}}$   $\boxed{B = -\frac{1}{5}}$

**RUBRIC**

+4 for specifying the correct form for the partial fraction:  $A/(x + 1) + (Bx + C)/(x^2 + 4)$.

+2 for multiplying both sides by the denominator:  $(x + 1)(x^2 + 4)$.

+3 for correctly solving for the unknowns  A,  B  and  C.

+1 for displaying the correct answer

(10 points) 6. Assume that the following sequence is arithmetic and find the sum using appropriate formulas: $7 + 3 - 1 - 5 - \ldots -201$.

$$a_1 = 7 \qquad d = 3 - 7 = -4$$

$$a_n = a_1 + (n-1)d$$

$$-201 = 7 + (n-1)(-4)$$

$$-208 = -4(n-1)$$

$$52 = n-1$$

$$n = 53$$

$$S_n = \frac{a_1 + a_n}{2} \cdot n$$

$$S_{53} = \frac{7 + (-201)}{2} \cdot 53 = \boxed{-5141}$$

**RUBRIC**

+1 for correctly identifying the first term
+1 for correctly finding the common ratio d = -4
+4 for correctly finding the n, that is
    +1 for correct formula for the n-th term
    +3 for correct calculations leading to   n = 53
+3 for correctly finding the sum term, that is
    +1 for correct formula of the sum
    +1 for correctly substituting the terms into the sum
    +1 for correct calculations
+1 for correct final answer

Note.
- If a student does not use the formula for the sum, but finds the sum correctly (by writing and adding the terms) award 3pt for the whole question  (the question specifically asks for the formula)
- If a student does not use the formula for the sum, only writes the terms and add the terms, but the final answer is not correct, award at most 1pt (for arithmetic) for the whole question

(10 points) 7. Let $A = \begin{bmatrix} 7 & -9 & 4 \\ 0 & 3 & -1 \\ 2 & -2 & 1 \end{bmatrix}$. Find the inverse $A^{-1}$.

$$\left[\begin{array}{ccc|ccc} 7 & -9 & 4 & 1 & 0 & 0 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 2 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 1 & 0 & -3 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 2 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \approx \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 1 & 0 & -3 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 0 & 4 & -1 & -2 & 0 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 1 & 0 & -3 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -2 & -1 & 7 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 1 & 0 & -3 \\ 0 & 1 & 0 & -2 & -1 & 7 \\ 0 & 3 & -1 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -3 & 1 & 1 & 0 & -3 \\ 0 & 1 & 0 & -2 & -1 & 7 \\ 0 & 0 & -1 & 6 & 4 & -21 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 7 & 4 & -24 \\ 0 & 1 & 0 & -2 & -1 & 7 \\ 0 & 0 & 1 & -6 & -4 & 21 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -3 \\ 0 & 1 & 0 & -2 & -1 & 7 \\ 0 & 0 & 1 & -6 & -4 & 21 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 1 & 1 & -3 \\ -2 & -1 & 7 \\ -6 & -4 & 21 \end{bmatrix}$$

**RUBRIC**

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

+2 for setting up (all or nothing)

+6 for correct row operations
  • Deduct points for arithmetic errors with correct row operations
  • Only give points for correct row operations
+1 for arriving with the identity matrix on the left side
+1 for identifying the inverse

(10 points) 8. Solve the equation: $\log_8 x^3 - \log_8 2x = 1$.

$$\log_8 x^3 - \log_8 2x = 1$$

$$\log_8 \frac{x^3}{2x} = 1$$

$$\frac{x^3}{2x} = 8^1 = 8$$

$$x^3 = 16x$$

$$x^3 - 16x = 0$$

$$x(x^2 - 16) = 0$$

$$x(x-4)(x+4) = 0$$

$x \neq 0$  (x = 4)  x = -4

**RUBRIC**

+2 for $log_8 x^3 - log_8 2x = log_8 \frac{x^3}{2x}$

+2 for $\frac{x^3}{2x} = 8$

+2 for correctly manipulating the equation and arriving with $x^3 - 16x = 0$
+2 for correctly factoring the equation to $x(x-4)(x+4) = 0$
+1 for identifying potential solutions as 0, -4, 4
+1 for concluding that the only answer is $x = 4$

Note:
   If a student starts with equation like $x^3 - 2x = 1$ the student can only receive 1pt for the whole question

(10 points) 9. A radioactive isotope that decays according to the function $A(t) = A_0 e^{-0.023t}$, where $A$ is the initial amount present and $A$ is the amount present at time $t$ (in days). Assume that a scientist has a sample of 90 grams of the isotope, when will 60 grams of the isotope be left? **Round your answer to two decimal places.**

$$60 = 90 e^{-0.023t}$$

$$\frac{2}{3} = e^{-0.023t}$$

$$\ln \frac{2}{3} = -0.023t$$

$$t = \frac{\ln \frac{2}{3}}{-0.023} \approx \boxed{17.63}$$

**RUBRIC**

+1 for correctly identifying $A = 60$, and $A_0 = 90$ (does not need to be stated explicitly)
+3 for correctly isolating for $e^{-0.023t}$, before taking ln of both sides, that is
    +1 for dividing both sides by 90
    +2 for arriving with $\frac{2}{3} = e^{-0.023t}$

+1 for recognizing the need to take ln of both sides
+1 for applying the properties of logarithms to correctly bring the exponent down
+1 correctly isolating $t$ (that is for dividing by the coefficient standing by $t$)
+2 for correct calculation
+1 for correct answer

**Note.**
- If a student does not isolate for $e^{-0.023t}$ before taking ln of both sides, but uses properties of logarithms correctly and arrives with correct answers, award full credit

- If a student does not isolate for $e^{-0.023t}$ before taking ln of both sides and that results with the wrong answer, the student is only receive maximum of 4pt for the whole questions (The points distribution is a s follow: 1pt for identifying $A$, $A_0$ correctly, 2pt for some correct arithmetic and one point for demonstrating the ability to round an answer to two decimal places )

(10 points) 10. Use the Binomial Theorem to determine which term of the expansion $(1 - 3b^2)^7$ contains $b^6$, find it and simplify it.

$$(k+1)\text{th term} = \binom{7}{k}(1)^{7-k}(-3b^2)^k \qquad \binom{7}{3}(1)^4(-3b^2)^3$$

$$b^{2k} = b^6$$

$$2k = 6$$

$$k = 3$$

$$= \frac{7!}{4!\,3!}(-27)b^6$$

$$= \frac{5\cdot 6\cdot 7}{1\cdot 2\cdot 3}\cdot(-27)b^6$$

$$= \boxed{-945\,b^6}$$

## RUBRIC

**+4pt** for correctly identifying the third term
**+6pt** for correctly calculating the term, that is:
    +1 for correctly setting up the term
    +1 for correct calculation of C(7,3)   (calculator is accepted, that is student does not
                                                    need to show factorials)
    +1 for correct exponents
    +2 for correctly performing the cube:   $(-3b^2)^3$ as $-27b^6$
    +1 for correct answer

**Note.**
- Students may choose to expand the binomial (using Binomial Thm or Pascal's Triangle) and identify the term from there, if that is done correctly award all points. If errors are encounter, award points as outlined above (for setting up, for the coefficients, for exponents, for performing the exponents correctly, for calculating the coefficients correctly)
- If a student expands the binomial up to the fifth term and notices that it is the term needed, award all points if the term and the expansion are correct
- Deduct a point if the needed term is calculated correctly and the answer is correct, but the whole expansion has one or two small arithmetic errors
- Deduct 2pt if multiple errors are encountered in the expansion, but the needed term is calculated correctly and the answer is correct
- If a student multiplies out (foil) the binomial seven times and get the correct answer, award no more than 4pt
- If a student multiplies out (foil) the binomial seven times and does not get the correct answer, award no more than 2pt

(10 points) 11. Solve the system of nonlinear equations: $\begin{cases} x - 2y = 5 \\ x^2 + 3y^2 = 13 \end{cases}$

$\begin{cases} x - 2y = 5 \\ x^2 + 3y^2 = 13 \end{cases}$ $\qquad$ $\begin{cases} x = 2y + 5 \\ (2y+5)^2 + 3y^2 = 13 \end{cases}$

$4y^2 + 20y + 25 + 3y^2 = 13$

$7y^2 + 20y + 12 = 0$

$(7y + 6)(y + 2) = 0$

$y = -\dfrac{6}{7} \qquad y = -2$

$\boxed{y = -2 \qquad x = 1}$

$\boxed{y = -\dfrac{6}{7} \qquad x = \dfrac{23}{7}}$

**RUBRIC**

+1 for isolating one variable from the first equation
+6 for correct calculations of the first variable; distribute the points as follow:
    +1 for correctly substituting the one variable into the second equation
    +2 for correctly performing the square
    +1 for correct calculations and arriving at correct equation with one variable
    +1 for correctly factoring the quadratic equation leading
    +1 for correct values of the first variable
+2 for correctly solving for the second variable
+1 for correctly stating the answer, either as ordered pair or clearly communicating the solution
    as pairs of x and y

(10 points) 12. Solve inequality. Write your answer in interval notation. $|3 - 2x| + 1 < 5$.

$$|3 - 2x| + 1 < 5$$

$$|3 - 2x| < 4$$

$$-4 < 3 - 2x < 4$$

$$-7 < -2x < 1$$

$$\frac{7}{2} > x > -\frac{1}{2}$$

$$\left( -\frac{1}{2}, \frac{7}{2} \right)$$

**RUBRIC**

+1 for subtracting 1 from both sides
+4 for correctly transposing the absolute value statement into a system of inequalities

- Note: Student may choose to write two separate statements
$$3 - 2x < 4 \text{ and } 3 - 2x > -4 \quad \text{This is fine.}$$
- Note: Students may choose to solve by squaring both sides of the inequality. If this method is done mathematically correct, the student should be awarded all the points in this section.
- Any wrong answer that shows inability to translate this absolute value inequality will receive none of these points

+1 for subtraction 3 from both sides and arriving at $-7 < -2x < 1$

+2 for dividing both sides by a negative and switching the inequality

+1 for arriving at $\frac{7}{2} > x > -\frac{1}{2}$
+1 for proper interval notation

Note:
- For students that do not begin by isolating the absolute value on one side of the inequality, the student is only eligible to receive no more than 4 points (that is, 1pt for interval notation, 2pt for arithmetic and up to 1 additional point if the grader feels the student demonstrates some understanding of transposing absolute value statements)

(10 points) 13. Using an algorithm called insertion sort, a common minicomputer can sort $N$ numbers from least to greatest in $t$ milliseconds where $t = 0.00339N^2 + 0.001143N - 5.95$. How many numbers can the minicomputer sort in 1 second (1000 milliseconds)? **Approximate your answer to the nearest integer.**

$$0.00339N^2 + 0.001143N - 5.95 = 1000$$

$$0.00339N^2 + 0.001143N - 1005.95 = 0$$

$$N = \frac{-0.001143 \pm \sqrt{(0.001143)^2 - 4 \cdot 0.00339 \cdot (-1005.95)}}{2 \cdot 0.00339}$$

$$N \approx 545 \qquad N \approx -545$$

**RUBRIC**

+1 for setting up the equation
+1 for subtracting 1000 from both sides to create zero on the other side
+1 for recognizing the need for quadratic formula or completing the square
+1 for correct substituting the numbers to the quadratic formula
+4 correct calculations

- Award +1 if small arithmetic errors are encountered but work is shown
- Award +0 if multiple arithmetic errors and/or major order of operations problems are made
- Award +0 if quadratic formula is recalled incorrectly

+1 for correctly calculating the roots as 545 and -545
+1 for concluding that the answer is 545

(10 points) 14. Let $f(x) = \frac{2x-1}{x+2}$.

(a) Find the inverse of the function $y = f(x)$.

$$y = \frac{2x-1}{x+2}$$

$$x(y+2) = 2y-1$$

$$y(x-2) = -2x-1$$

$$x = \frac{2y-1}{y+2}$$

$$xy + 2x = 2y-1$$

$$y = \frac{-2x-1}{x-2}$$

$$xy - 2y = -2x-1$$

$$f^{-1}(x) = \frac{-2x-1}{x-2} = \frac{2x+1}{2-x}$$

(b) Determine the range of the function $y = f(x)$.

$$R(f) = D(f^{-1}) = \left\{ y \mid y \neq 2 \right\}$$

**RUBRIC**

(a)  **Total of 6 points for part (a)**
   +1 for swapping x with y  and solving for y
   - A student may choose to keep solving for x without interchanging x and y
   +1 for multiplying by the denominator
   +1 for bringing the terms with y into one side
   - If a student is solving for x (without swapping x and y), then the point with apply for bringing the terms that contain x to one side
   +1 for factoring y   (or  x – if student is solving for x)
   +1 for dividing by the factor standing by y
   +1 for correct answer


(b)  **Total of 4 points for part (b)**
   +1 for noticing that range is the domain of the inverse (it does not need to be stated explicitly)
   +2 for restriction  $y \neq 2$  or  $x \neq 2$
   - If a student uses both $x \neq -2$, $x \neq 2$  award 1pt
   +1 for correct answer using correct notation, that is either as  $y \neq 2$
   or  $(-\infty, 2) \cup (2, \infty)$ or in set notation using the variable y

(10 points) 15. Let $f(x) = 2^{x+1} + 1$.
(a) Determine the domain.

$(-\infty, \infty)$

(b) Find all intercepts of its graph, if any.
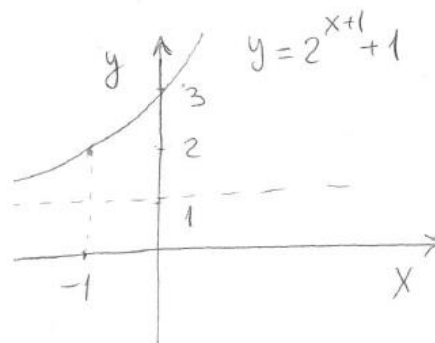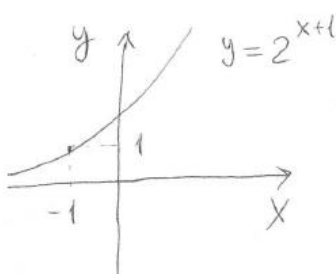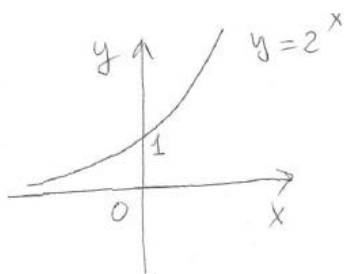
$y_{int} = (3, 0)$      $2^{x+1} + 1 = 0$           $x_{int} - none$

$2^{x+1} = -1$    No solution

(c) Find all asymptotes of its graph.

H.A.   $y = 1$

(d) Graph the function $f(x)$ using transformations. Start with graphing $g(x) = 2^x$ and show all steps.



## RUBRIC

(a) **Total of 2 points for part (a)**
   +2 for correct domain

(b) **Total of 3 points for part (b)**
   +2 for correct y-int (either as coordinate points (3,0) or as y=3)
   +1 for indicating there is no x-int, or simply not making any statement about the x-int
   - If coordinates are confused and students states (0,3), award 1pt

(c) **Total of 2 points for part (c)**
   +2 for horizontal asymptote y=1
   - A student may simply state: y=1, award full points
   - If a student says just: 1 without indicating what asymptote it is: horizontal or vertical, award 1pt

(d) **Total of 3 points for part (d)**
   +1 for maintaining the shape of the exponential function
   +1 for horizontal shift one unit to the left
   +1 for vertical shift one unit up
   - The last two steps might be combine as one, if clearly communicated, award full points