Math 1050 College Algebra
Rubric for Sample Final Exam 1

General Guidelines

- Only perfectly completed problems should receive full credit. If the solution contains an arithmetic error, at least one point must be deducted
- Only award and deduct an integer number of points

(6 points) **Question 1**. True or false.

(F) $\log_3 \frac{x^2+y}{z} - 2\log_3 x + \log_3 y - \log_3 z$ for all $x > 0$, $y > 0$, and $z > 0$.
(F) The graphs of $f(x) - x^2$ is symmetric about x –axis.
(T) The solution of the inequality $|x - 1| \leq 2$ is $[-1, 3]$.

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For each, award 2 points for the correct response and 0 points for wrong or blank answers

Question 2.

(9 points) 2. Fill in the blank.

(a) $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix}$

(b) The solution of $3^{2x} = \sqrt[3]{3}$ is $x =$ 1/6

(c) If $f(x) = 3x + 1$, then its inverse $f^{-1}(x) =$ $\frac{X-1}{3}$

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Award 3pt for each correct answer

(a) +3 for correct answer
- Award 2pt is one of the numbers in the matrix is incorrect
- Award 1pt if two of the numbers are incorrect
- Award 0pt if more than two numbers in the matrix are incorrect

(b) +3 for correct answer

(c) +3 for correct answer
- Award 1pt for answer like $\frac{x+1}{3}$

(10 points) 3. Solve the inequality. Write your answer in interval notation. $\frac{2x-1}{x+2} \leq 1$.

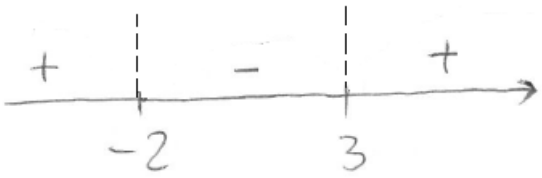
$\frac{2x-1}{x+2} \leq 1$

$\frac{2x-1}{x+2} - 1 \leq 0$

$\frac{2x-1-1(x+2)}{x+2} \leq 0$

$\frac{2x-1-x-2}{x+2} \leq 0$

$\frac{x-3}{x+2} \leq 0$



$(-2, 3]$

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+1 for subtracting 1 from both sides to create zero on one side
+1 for arriving at $\frac{2x-1-(x+2)}{x+2} \leq 0$, or something equivalent that shows correct common denominator
+1 for correctly distributing minus and arriving at $x - 3$ in the numerator
+1 for identifying critical points -2 and 3
- Do not need to be labeled as such to receive the points

+1 for recognizing the need to evaluate the behavior of the function in different regions
+2 for correctly implementing a method to evaluate the behavior of the function in different regions (using a number line, logical connectives, or graphing)
- Award +1 if a student makes a small error in work
  Student must show and communicate clearly to be awarded the partial point
- Award +0 if a student shows no ability to carry out the chosen method

+1 for correctly identifying the regions for which the function is less than or equal to zero
+1 for correct end points (that is -2 is not included and 3 is included in the solution)
+1 for correct interval notation
- Award based on the answer from previous step whether or not that is the correct interval

Note:
- If a student begins by "cross multiplying", award only one point for correct interval notation
- If a student avoids checking regions, the student is only eligible for maximum of 4pt (1pt for correct interval notation, 1pt for correct end points and 2pt for correct calculations)

(15 points) 4. Consider the rational function $f(x) = \frac{x^2}{x+1}$.

(a) State its domain.

$D = \{x \mid x \neq -1\}$

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(a) **Total of 2 points for part (a)**

+2 for $x \neq -1$

- Domain may be given in set notation, interval notation, or just by identifying the restrictions.
- Award 1pt if other restrictions are also given

(b) Find all intercepts of its graph, if any.

$x_{int} = y_{int} = (0, 0)$

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(b) **Total of 2 points for part (b)**

- Student may simply say (0,0) , then award 2pt
- If a student says only x=0, award 1pt
- If a student says only y=0, award 1pt
- If a student writes only 0, award 1pt
- Take away a point if another intercept is also given

(c) Find all asymptotes of its graph.

V.A. $x = -1$, O.A: $y = x - 1$

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(c) **Total of 3 points for part (c)**

+1 for V.A.

+2 for O.A.

(d) Determine whether its graph crosses a non-vertical asymptote. **Justify your answer.**

$\frac{x^2}{x+1} = x-1$ $\quad x^2 = (x+1)(x-1)$ $\quad x^2 = x^2 - 1$ $\quad 0 \neq -1$ Does not Cross

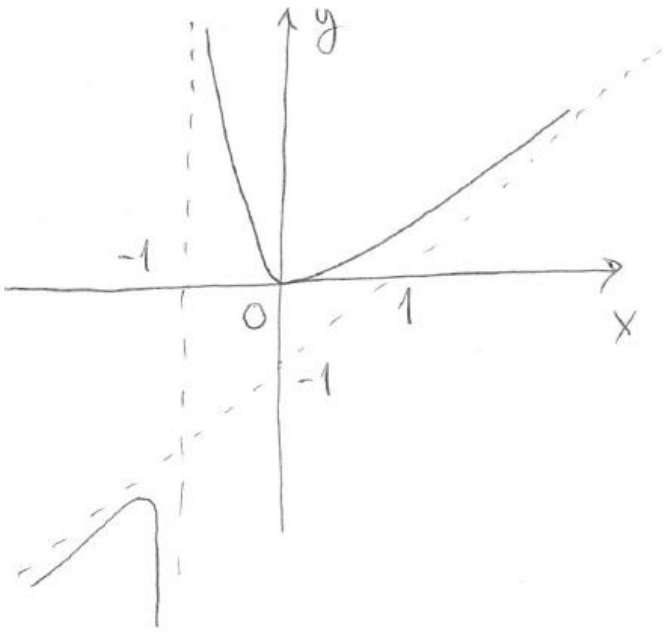
RUBRIC

(d) **2 points total for part (d)**

+1 for $\frac{x^2}{x+1} = x - 1$

+ 1 for concluding that the graph does not crosses the asymptote

(e) Use the above information and other appropriate points to draw its graph.



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(e) **Total of 6 points for part (e)**

+1 for showing the x-intercepts

+1 for plotting the asymptotes

+1 for correct behavior in Region 1

+1 for correct behavior in Region 2

+2 for showing correctly asymptotic behavior

Note. If more than two pieces of the graph are drawn, deduct 2pt from part (e)

(10 points) 5. Find the partial fraction decomposition of the rational expression.

$$\frac{3x}{(x+2)(x-1)}$$

(10 points) 6. Find a_8 of the geometric sequence given that $a_3 = 0.5$ and $a_4 = 8$.

$$\begin{cases} a_3 = a_1 r^2 = \frac{1}{2} \\ a_4 = a_1 r^3 = 8 \end{cases} \quad \begin{cases} a_1 = \frac{1}{2r^2} \\ \frac{1}{2r^2} \cdot r^3 = 8 \end{cases} \quad \begin{cases} a_1 = \frac{1}{2r^2} \\ \frac{r}{2} = 8 \end{cases} \quad a_1 = \frac{1}{2 \cdot 16^2} \quad r = 16$$

$$a_8 = a_1 r^7 = \frac{1}{2 \cdot 16^2} \cdot 16^7 = \frac{16^5}{2} = 524,288$$

RUBRIC

+2 for finding the common ratio correctly r = 16
+3 for correctly finding the first term
+1 for using the formula for the n-th term
+3 for correctly finding the eight term
+1 for correct final answer

Note. If a student does not use the n-th term formula, but clearly communicates how he/she found the first term correctly and later the eight term correctly, award full points.

(10 points) 7. (a) Evaluate the determinant of the matrix $A = \begin{bmatrix} 0 & -1 & 4 \\ 2 & 1 & -1 \\ 2 & -2 & 0 \end{bmatrix}$.

$$\det A = 0 \cdot \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 2 & -1 \\ 2 & 0 \end{vmatrix} + 4 \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = 0 + 1 \cdot 2 + 4 \cdot (-6)$$

$$= -22$$

(b) How does the determinant of the matrix A will change if we add the first row to the second one and replace it: $\begin{bmatrix} 0 & -1 & 4 \\ 2 & 0 & 3 \\ 2 & -2 & 0 \end{bmatrix}$

$$\begin{vmatrix} 0 & -1 & 4 \\ 2 & 0 & 3 \\ 2 & -2 & 0 \end{vmatrix} = 0 \cdot \begin{vmatrix} 0 & 3 \\ -2 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 2 & 3 \\ 2 & 0 \end{vmatrix} + 4 \begin{vmatrix} 2 & 0 \\ 2 & -2 \end{vmatrix}$$

$$= 1 \cdot (-6) + 4 \cdot (-4) = -6 - 16 = -22$$

RUBRIC

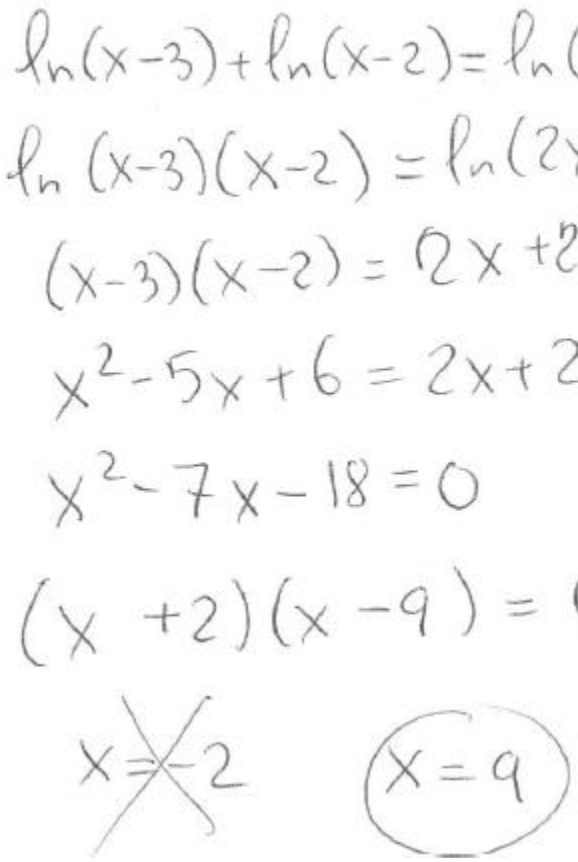
(a) Total of 6pt for part (a)

+1 for choosing correct method to evaluate the determinant
+4 for correctly carried out the chosen method
Deduct a point for small arithmetic error
Deduct 2pt for several small arithmetic errors
Deduct 3pt for severe errors
Award 0pt for this step if a student shows no ability to find the determinant
+1 for correct answer

(b) Total of 4pt for part (b)

- Students may choose to use properties of determinants and simply state that the determinant does not change, award all points for such statement
- Students may choose to find the determinant in part (b), if that is done correctly award all points for part (b)
- If a student calculates the determinant in part (b), but makes error(s)
  subtract 1pt for small errors,
  subtract 2 -3pt for several errors
- If a student calculates the determinant in part (a) incorrectly and in part (b) states that the determinant is the same (without calculating it), award all points for part (b)

(10 points) 8. Solve the equation. $\ln(x-3)+\ln(x-2)=\ln(2x+24)$



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+2 for $\ln(x-3)+\ln(x-2)=\ln(x-3)(x-2)$
+1 for equating the arguments of the logarithms, that is (x-3)(x-2) = 2x + 24
+1 for correctly multiplying $(x-3)(x-2)=x^2-5x+6$
+1 for arriving at $x^2-7x-18=0$
+2 for factoring to $(x+2)(x-9)=0$
+1 for arriving at $x = -2$, $x = 9$
+1 for checking the potential solutions (does not need to be stated explicitly)
+1 for concluding that the only answer is $x = 9$

Note: If the student is not able to correctly manipulate the equation to get to a quadratic to solve, points can only be awarded if the student correctly factors something (+1) and if the student checks potential solutions (+1).

- If a student starts with equation like: x-3 + x-2 = 2x + 24, the student can only receive 1pt for the whole question

(10 points) 9. A radioactive isotope has a half-life of 16 days. What is its relative rate of decay k? ($m(t) = m_0 e^{kt}$.) **Round only your final answer to two decimal places.**

$0.5 = e^{16k}$

$\ln 0.5 = 16k$

$k = \frac{\ln 0.5}{16} \approx -0.04$

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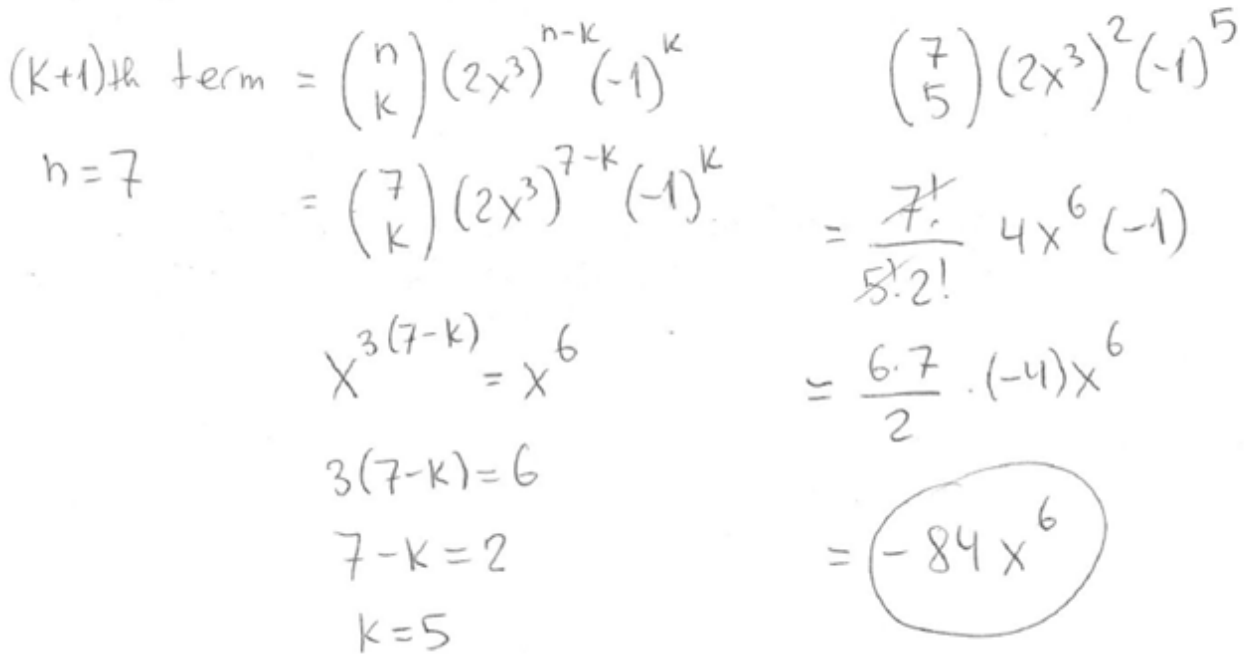
(10 points) 10. Use the Binomial Theorem to determine which term of the expansion $(2x^3 - 1)^7$ contains x^6, find the term and simplify it.



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+4pt for correctly identifying the seventh term

+6pt for correctly calculating the term, that is:

+1 for correctly setting up the term

+1 for correct calculation of C(7,5) = 21 (calculator is accepted, that is student does not need to show factorials)

+1 for correct exponents

+1 for correctly performing the square: $(2x^3)^2$ as $4x^6$

+1 for correct treatment of the minus

+1 for correct answer

Note.

- Student may choose to expand the binomial (using Binomial Thm or Pascal's Triangle) and identify the term from there, if that is done correctly award all points. If errors are encounter, award points as outlined above (for setting up, for the coefficients, for exponents, for performing the exponents correctly, for calculating the coefficients correctly)
- If a student expands the binomial up to the fifth term and notices that it is the term needed, award all points if the term and the expansion are correct
- Deduct a point if the needed term is calculated correctly and the answer is correct, but the whole expansion has one or two small arithmetic errors
- Deduct 2pt if multiple errors are encountered in the expansion, but the needed term is calculated correctly and the answer is correct
- If a student multiplies out (foil) the binomial seven times and get the correct answer, award no more than 4pt
- If a student multiplies out (foil) the binomial seven times and does not get the correct answer, award no more than 2pt

(10 points) 11. Solve the system of nonlinear equations: $\begin{cases} x+y=-3 \\ x^2+y^2=17 \end{cases}$

$\begin{cases} x+y=-3 \\ x^2+y^2=17 \end{cases}$ $\begin{cases} y=-3-x \\ x^2+(-3-x)^2=17 \end{cases}$ $\begin{cases} y=-3-x \\ x^2+9+6x+x^2=17 \end{cases}$

$2x^2+6x-8=0$

$2(x^2+3x-4)=0$

$2(x+4)(x-1)=0$

$x=-4 \quad x=1$

$x=-4 \quad y=1$

$x=1 \quad y=-4$

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+1 for isolating one variable from the first equation
+6 for correct calculations of the first variable; distribute the points as follow:
- +1 for correctly substituting the one variable into the second equation
- +2 for correctly performing the square
- +1 for correct calculations and arriving at correct equation with one variable
- +1 for correctly factoring the quadratic equation
- +1 for correct values of the first variable

+2 for correctly solving for the second variable
+1 for correctly stating the answer, either as ordered pairs or clearly communicating the solution as pairs of x and y values

(10 points) 12. Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x+1}$.

(a) Find the composition $(f \circ g)(x)$ and simplify it.

$(f \circ g)(x) = f(g(x)) = \frac{1}{\frac{1}{x+1}} = x+1$

(b) Determine the domain of the function $(f \circ g)(x)$ and state the answer in the set notation.

$D(g) = \{x \mid x \neq -1\}$

$D(f) = \{x \mid x \neq 0\}$

$D(f \circ g) = \{x \mid x \neq -1\}$

$g(x) = 0$

$\frac{1}{x+1} = 0$

$1 \neq 0$ No solution

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(a) **Total of 5 points for part (a)**

+2 for correctly substituting function g into f
+2 for correctly simplifying it
+1 for arriving at x + 1

(b) **Total of 5 points for part (b)**

+2 for domain of g, $x \neq 1$
+1 for noting that there is no restrictions form function f (does not need to be stated explicitly)
+1 for correct final answer, that is $\{x|x \neq 1\}$
+1 for set notation

- If a student has two restrictions in the domain, such as $x \neq 1$ and $x \neq 0$, deduct 2pt for part (b)

(10 points) 13. Let $f(x) = \log_2(x-1) + 1$.

(a) Determine the domain.

$x-1>0, \quad x>1, \quad D(f)=\{x \mid x>1\}$

(b) Find all intercepts of its graph.

y_{int} - none

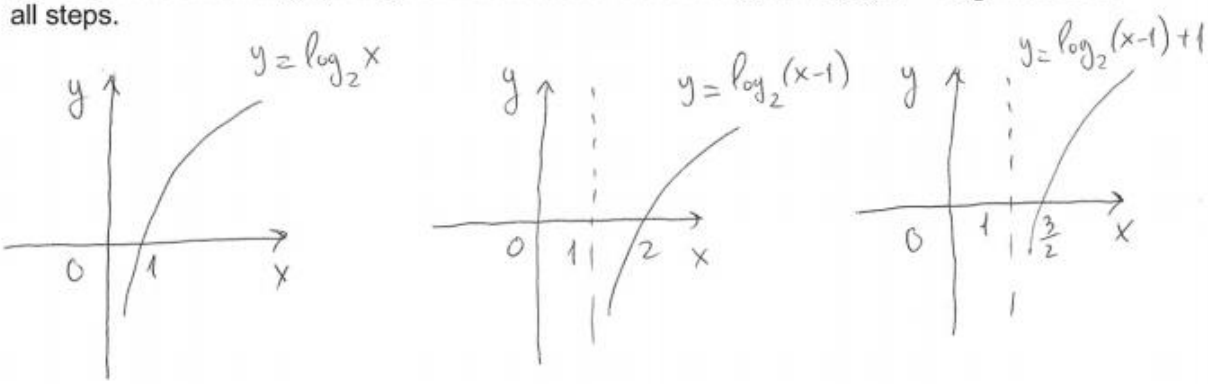
$\log_2(x-1)+1=0 \quad \log_2(x-1)=-1 \quad x-1=2^{-1}=0.5 \quad x=1.5$

$x_{int}=(1.5,0)$

(c) Find all asymptotes of its graph.

V.A. $x=1$

(d) Graph the function $f(x)$ using transformations. Start with graphing $g(x) = \log_2 x$ and show all steps.



RUBRIC

(a) **Total of 2 points for part (a)**

+2 for correct restriction $x > 1$

- Deduct a point if more than one restriction is stated
- Note. Domain may be given in any form (in set notation, interval notation, or just by identifying the restriction)

(b) **Total of 3 points for part (b)**

+1 for clearly communicating that there is no y-int. or making no statement about y-int., only because there is no y-int

+2 for correctly calculating the x-int. (1pt for correct setting and 1pt for correct calculation leading to the correct answer)

Note. If a student simply says: (3/2,0) (or x-int. at 3/2), award full points for part (b)

(c) **Total of 2 points for part (c)**

+2 for correctly stating the vertical asymptote

- If a student simply says: x=1, award all points for part (c)
- If a student just says: 1 (without specifying if the asymptote is vertical or horizontal), award 1pt

- If a student write horizontal asymptote at 1 (or H.A. x = 1), award 1pt

(d) **Total of 3 points for part (d)**

+1 for maintaining the shape of the logarithmic function
+1 for horizontal shift 1 unit to the right
+1 for vertical shift 1 unit up