Extra Examples for FINAL 2019:

1. **(6 points)** Consider the function $g(x) = \frac{3x^2+2}{x^2+2x-8}$. **Write your answer(s) in equation form.**
   a. The vertical asymptote(s), if any, of the function $g(x)$ is (are)

   $x = 2$, $x = -4$

   $\frac{3x^2+2}{(x-2)(x+4)}$

   b. The non-vertical asymptote(s), if any, of the function $g(x)$ is (are)

   $y = 3$

   $\frac{3x^2+2}{x^2+2x-8}$

   same degree = divide leading coefficients

2. **(3 points)** Consider the system of linear equations: $\begin{cases} 2x+4y-z=3 \\ -x+y-4z=0 \\ 4x+7y+z=-1 \\ x+y+6z=10 \end{cases}$ What is a valid first step to find the solution of the given system of linear equations using matrices (row operations)?

   set up the matrix!

   $$\left[\begin{array}{ccc|c} 2 & 4 & -1 & 3 \\ -1 & 1 & -4 & 0 \\ 4 & 7 & 1 & -1 \\ 1 & 1 & 6 & 10 \end{array}\right]$$

3. **(3 points)** Let $f(x) = x^4 - 8$. What is the inverse f^{-1}?

   $y = x^4 - 8$

   $x = y^4 - 8$

   $+8 \quad +8$

   $\sqrt[4]{x+8} = \sqrt[4]{y^4}$

   $y = \sqrt[4]{x+8}$

   $f^{-1}(x) = \sqrt[4]{x+8}$

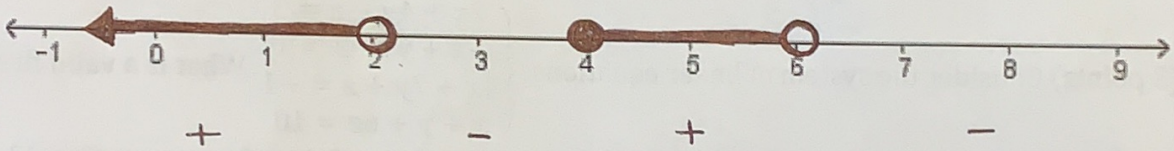
4. **(2 points)** Write 2 correct ways to write the intervals in **interval notation** marked on the number line below.

$(-\infty, 0) \cup [4, +\infty)$

$(-\infty, 0)$ OR $[4, +\infty)$

5. **(2 points)** The rational expression $\frac{4-x}{x^2-8x+12}$ has critical numbers at $x = 2, x = 4$ and $x = 6$. $(x-6)(x-2)$

Find the solution to the inequality $\frac{4-x}{x^2-8x+12} \geq 0$. Graph solution on number line.



6. **(4 points)** Consider the function: $f(x) = log_2(x+3)$ left 3 →

a. What is the domain of f?

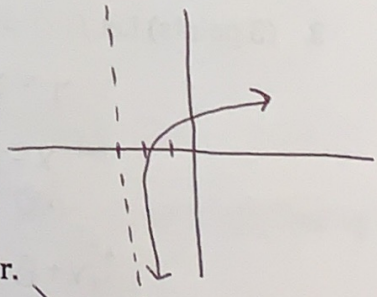
$x+3>0$

$x>-3$

$x > -3$

or

$(-3, +\infty)$



has an x-int. & y-int

b. What is the intercept of f? **Write your answer as an ordered pair.**

$0 = \log_2(x+3)$

$2^0 = x+3$

$2^0 - 3 = x$

$-2 = x$

$(-2, 0)$

x-int

$y = \log_2(0+3)$

$y = \log_2(3)$

$y = \frac{\log(3)}{\log(2)} = 1.58$

$(0, 1.58)$

y-int

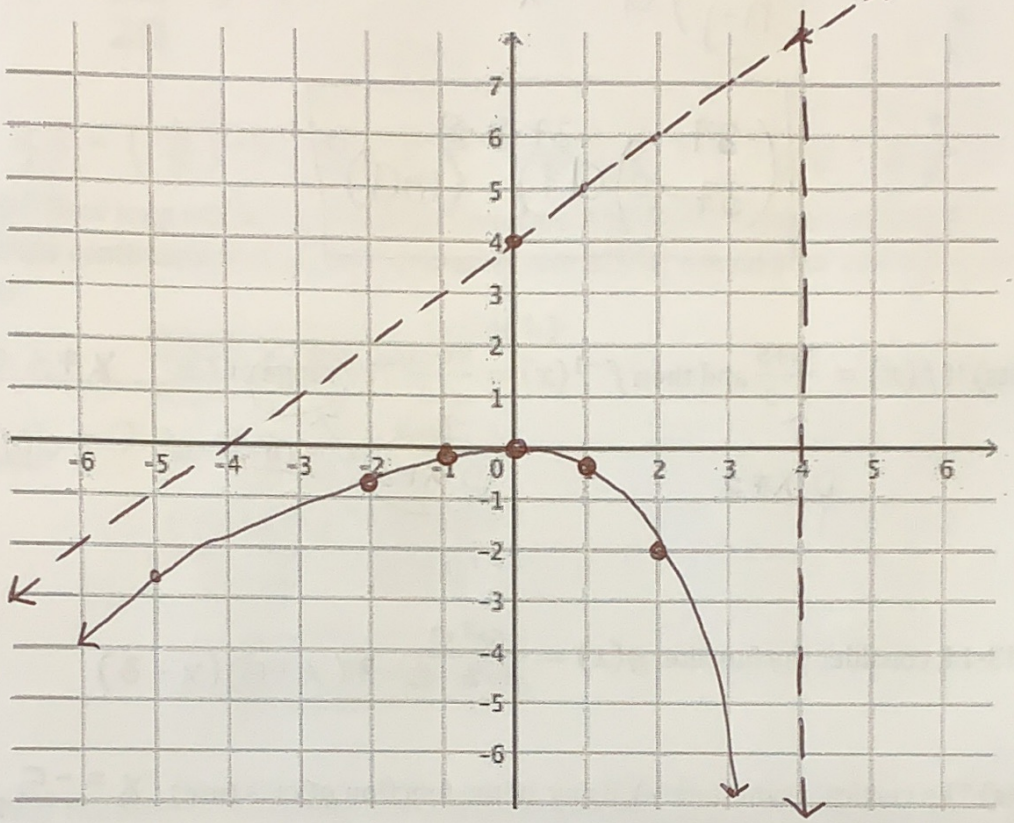
7. **(9 points)** Graph the rational function $f(x) = \frac{x^2}{x-4}$. Your graphs should clearly show and label all x and y-intercept(s) and all asymptotes.

V.A. : $x=4$

oblique:

$$\begin{array}{r|ccc} 4 & 1 & 0 & 0 \\ & \downarrow & 4 & 16 \\ \hline & 1 & 4 & 16 \end{array} \quad R$$

$y = x+4$



x	y
-2	-0.7
-1	-0.2
0	0
1	-0.3
2	-2
5	25
6	18
8	16

8. **(2 points)** Multiply: $\begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 9 & -3 \\ 6 & 1 \end{bmatrix}$

$$\begin{bmatrix} (-1)(9)+(4)(6) & (-1)(-3)+(4)(1) \\ 2(9)+(3)(6) & 2(-3)+(3)(1) \end{bmatrix} = \begin{bmatrix} 15 & 7 \\ 36 & -3 \end{bmatrix}$$

9. **(2 points)** The graph of the function $f(x) = 5x^8 + 6x^4 - 3$ has at most 7 turning points.

10. **(3 points)** If $\log_b x^4 = 7$, evaluate $\log_b x$.

$\log_b x = \frac{7}{4}$

$\frac{4 \log_b x}{4} = \frac{7}{4}$

11. **(3 points)** Write an expression to find the term containing a^8 in the expansion $(6a-1)^{37}$ **Do not simplify.**

j x a n

$$\binom{n}{n-j} a^{n-j} x^j$$

$$\boxed{\binom{37}{37-8}(-1)^{37-8}(6a)^8}$$

12. **(3 points)** If $f(x) = \frac{3x+5}{x-3}$ and then $f^{-1}(x) = \frac{3x-5}{x-3}$. The **range** of f is $x \neq 3$ or $y \neq 3$.

D: $x \neq 3$ (for f)

D: $x \neq 3$ (for f^{-1})

range of f = domain of f^{-1}

For problems 13-14, consider the function $g(x) = \frac{4x^2+3}{x^2+2x-15}$ → $(x+5)(x-3)$

13. **(3 points)** The vertical asymptote(s), if any, of the function $g(x)$ is (are) $x = -5, x = 3$.
Write your answer(s) as equation(s).

14. **(3 points)** The non-vertical asymptote(s), if any, of the function g(x) is (are) $y = 4$.
Write your answer(s) as equation(s).

15. **(3 points)** Given the table:

x	-3	-2	-1	0	1	2	3
$f(x)$	-7	-5	-3	-1	3	5	7
$g(x)$	8	3	0	-1	0	3	8

Evaluate $(g \circ f)(-1)$ 8.

$g(f(-1)) = g(-3) = 8$

16. **(3 points)** The maximum value (y) of the function $f(x) = -x^2 - 3x + 7$ is: $y =$ $\frac{37}{4}$ or 9.25

($a = -1$, $b = -3$)

$$x = \frac{-b}{2a} \Rightarrow x = \frac{-(-3)}{2(-1)} = \frac{3}{-2}$$

$$y = -\left(-\tfrac{3}{2}\right)^2 - 3\left(-\tfrac{3}{2}\right) + 7 = -\frac{9}{4} + \frac{9}{2} + 7 = -\frac{9}{4} + \frac{18}{4} + \frac{29}{4} = \frac{37}{4} \text{ or } 9.25$$

17. **(3 points)** How long will it take an initial investment of \$1000 to be worth \$6500 if the investment **compounds continuously** at an annual interest rate of 4%? Round your answer to the nearest tenth of a year.

$$\frac{6500}{1000} = \frac{1000e^{0.04(t)}}{1000}$$

$$6.5 = e^{0.04(t)}$$

$$\frac{\ln(6.5)}{0.04} = \frac{0.04(t)}{0.04}$$

$t = 46.8$ yrs

18. **(4 points)** The form of the partial fraction decomposition of the rational function $f(x) = \frac{4x+1}{x(x+3)^2}$ is:

$$\frac{4x+1}{x(x+3)^2} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

19. **(4 points)** Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & -4 & 3 \\ 2 & 2 & 1 & -1 \end{bmatrix}$. Some row operation(s) have been applied to A to obtain $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -5 & -8 & x & y \\ 2 & 2 & 1 & -1 \end{bmatrix}$. What are the values of x and y?

$R_2 = -4r_1 + r_2$

$x = -4(3) + (-4) = -16$

$y = -4(4) + 3 = -13$

$x = -16$ $y = -13$

20. (4 points) Find the remainder: $\frac{5x^2+7x-3}{x+1}$ **-5**

$$\begin{array}{r|rrr} -1 & 5 & 7 & -3 \\ & \downarrow & -5 & -2 \\ \hline & 5 & 2 & -5 \end{array}$$

-5 is the remainder; 5x + 2 is the quotient

21. (7 points) Assume the following sequence is arithmetic. Find the sum using appropriate formulas.

$$4 - 1 - 6 - 11 - 16 - \cdots - 126$$

① Find n: $a_n = a_1 + (n-1)d$

$-126 = 4 + (n-1)(-5)$

$-126 = 4 + 5 - 5n$

$-126 = 9 - 5n$

$-9 \quad -9$

$-135 = -5n$

$n = 27$

② Find sum: $S_n = \frac{n(a_n + a_1)}{2}$

$S_{27} = \frac{27(-126+4)}{2}$

$S_{27} = -1647$

22. (5 points) Let $f(x) = \frac{3x+5}{x-3}$. Find the inverse of f and simplify completely.

$y = \frac{3x+5}{x-3}$

$(y-3) \cdot x = \frac{3y+5}{y-3} \cdot (y-3)$ ← get rid of fraction

$x(y-3) = 3y+5$ ← distribute

$xy - 3x = 3y + 5$

$-3y \quad -3y$ ← move all y's on one side

$xy - 3y - 3x = 5$

$+3x \quad +3x$

$xy - 3y = 3x + 5$ ← Factor out the y's

$y(x-3) = 3x+5$

$\frac{y(x-3)}{x-3} = \frac{3x+5}{x-3}$

$y = \frac{3x+5}{x-3}$

$f^{-1}(x) = \frac{3x+5}{x-3}$