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6.8 Exponential Growth & Decay Models

Uninhibited Growth of Cells:

$N(t) = N_0 e^{kt}$ $k > 0$

$N(t)$ = # of cells after time
N_0 = initial # of cells
k = growth rate
t = time

ex. 1) Given $N(t) = 100e^{0.045t}$, where N is in grams & t is in days.

(a) determine initial amount of bacteria

when $t=0 \rightarrow N(0) = 100e^{0.045(0)} = \boxed{100 \text{ grams}}$

(b) what is the growth rate?

$k = 0.045 \rightarrow \boxed{4.5\%}$

(c) what is population after 5 days?

$N(5) = 100e^{0.045(5)} \approx \boxed{125.2 \text{ grams}}$

(d) How long will it take pop to reach 140 grams?

$\frac{140}{100} = \frac{\cancel{100} e^{0.045t}}{\cancel{100}}$

$1.4 = e^{0.045t}$

$\frac{\ln(1.4)}{0.045} = \frac{\cancel{0.045}t}{\cancel{0.045}}$

$t = \frac{\ln(1.4)}{0.045} \approx \boxed{7.5 \text{ days}}$

(e) what is the doubling time for the population?
$\uparrow$ = 200 grams

$\frac{200}{100} = \frac{\cancel{100} e^{0.045t}}{\cancel{100}}$

$2 = e^{0.045t}$

$\rightarrow \ln(2) = 0.045t$

$t = \frac{\ln(2)}{0.045} \approx \boxed{15.4 \text{ days}}$

ex.2) Bacterial Growth

(a) If N is the # of cells & t is time in hrs. express N as a function of t.

$$N(t) = N_0 e^{kt}$$

(b) if # of bacteria doubles in 3 hrs, find function that gives the # of cells.

*need to find k

$N(3) = 2N_0$ so $\frac{2N_0}{N_0} = \frac{N_0 e^{k(3)}}{N_0}$

$t = 3$

$2 = e^{k(3)}$

$\frac{\ln(2)}{3} = \frac{k(3)}{3}$ $\quad \underline{k = 0.23105}$

$$\boxed{\text{Function: } N(t) = N_0 e^{0.23105t}}$$

(c) time required to triple the colony?

$N = 3N_0$ $\quad \frac{3N_0}{N_0} = \frac{N_0 e^{0.23105t}}{N_0}$

$3 = e^{0.23105t}$

$\ln(3) = 0.23105t$

$t = \frac{\ln(3)}{0.23105} \approx \boxed{4.755 \text{ hrs}}$

(d) How long will it take to increase 4 times (double again)?

it doubled in 3 hrs so it will double a second time in 3 hrs. for a total of 6 hrs.

Radio Active Decay :

$$A(t) = A_0 e^{kt} \quad k<0$$

ex.3) Estimate Age of Ancient Tools : (Burned wood) (Stone tools)

* 1.67% of original carbon 14
* half life of carbon is 5730 yrs.

When was the tree cut & burned?

① Find k: we know $A(5730) = \frac{1}{2}A_0$ so

$$\frac{1}{2}A_0 = A_0 e^{k(5730)}$$

$$\frac{1}{2} = e^{5730k}$$

$$\ln\frac{1}{2} = 5730k \rightarrow k = \frac{\ln(\frac{1}{2})}{5730} \approx -0.000120968$$

② New Formula: $A(t) = A_0 e^{-0.000120968t}$

③ if 1.67% of carbon is present $A(t) = \underbrace{0.0167A_0}_{1.67\% \text{ of } A_0}$

$$\text{so } 0.0167A_0 = A_0 e^{-0.000120968t}$$

$$0.0167 = e^{-0.000120968t}$$

$$\ln(0.0167) = -0.000120968t$$

$$t = \frac{\ln(0.0167)}{-0.000120968} \approx \boxed{33{,}830 \text{ yrs}}$$

Newton's Law of Cooling :

$$u(t) = T + (u_0 - T)e^{kt}$$

$$k<0$$

$u(t)$ = ending temp

u_0 = initial temp

T = constant temp surrounding medium

ex.4) An object is heated to 100°C & then is allowed to cool in a room whose air temp is 30°C.

(a) if temp of the object is 80°C after 5 min when will temp be 50°C?

$T=30$ (need K)
$u_0=100$
$\left.\begin{array}{l} u=80 \\ t=5 \end{array}\right\}$ use to find K

$80=30+(100-30)e^{K(5)}$

$80=30+70e^{5K}$

$\frac{50}{70}=\frac{\cancel{70}e^{5K}}{70}$

$\frac{5}{7}=e^{5K}$

$\ln\left(\frac{5}{7}\right)=5K$

$K=\frac{\ln(5/7)}{5}$

≈ -0.0673

Now find t for when $u=50°C$

$50=30+(100-30)e^{-0.0673t}$

$20=70e^{-0.0673t}$

$\frac{2}{7}=e^{-0.0673t}$

$\ln(2/7)=-0.0673t$

$t=\frac{\ln(2/7)}{-0.0673}$

$t=\boxed{18.6\text{ min}}$

(b) Determine time before temp is 35°C

need t when $u=35°C$

$35=30+70e^{-0.0673t}$

$5=70e^{-0.0673t}$

$\frac{5}{70}=e^{-0.0673t}$

$\ln\left(\frac{5}{70}\right)=-0.0673t$

$t=\frac{\ln(5/70)}{-0.0673}\approx\boxed{39.2\text{ min}}$

Logistic Models:

$$P(t) = \frac{C}{1+ae^{-bt}}$$

P = population after time
C = carrying capacity
|b| = growth rate | decay rate

ex.5) $P(t) = \frac{230}{1+56.5e^{-0.37t}}$

(a) carrying capacity

$\boxed{230 \text{ fruit flies}}$

(b) Determine Initial population

$$P(0) = \frac{230}{1+56.5e^{-0.37(0)}} = \frac{230}{1+56.5} = \boxed{4}$$

(c) what is population after 5 days?

$$P(5) = \frac{230}{1+56.5e^{-0.37(5)}} \approx \boxed{23 \text{ fruit flies}}$$

(d) How long does it take for the population to reach 180?

$$180 = \frac{230}{1+56.5e^{-0.37t}}$$

$$\frac{180(1+56.5e^{-0.37t})}{180} = \frac{230}{180}$$

$$\cancel{1}+56.5e^{-0.37t} = 1.2778$$
$$\cancel{-1} \qquad\qquad -1$$

$$\frac{56.5e^{-0.37t}}{56.5} = \frac{0.2778}{56.5}$$

$$e^{-0.37t} = 0.0049$$

$$\frac{\ln(0.0049)}{-0.37} = \frac{\cancel{-0.37}t}{\cancel{-0.37}}$$

$\boxed{t = 14.4 \text{ days}}$

ex.6) $P(t) = \frac{100.3952}{1+0.0316e^{0.0581t}}$

(a) decay rate?

$0.0581 \rightarrow \boxed{5.81\%}$

(b) What is the % of wood products after 10 years?

$$P(10) = \frac{100.3952}{1+0.0316e^{0.0581(10)}} = \boxed{95\%}$$

(c) How long does it take for the % of remaining wood products to reach 50%?

$$50 = \frac{100.3952}{1+0.0316e^{0.0581(t)}}$$

$$\frac{\cancel{50}(1+0.0316e^{0.0581t})}{\cancel{50}} = \frac{100.3952}{50}$$

$$(\cancel{1}+0.0316e^{0.0581t}) = 2.0079$$
$$\cancel{-1} \qquad\qquad -1$$

$$\frac{\cancel{0.0316}e^{0.0581t}}{\cancel{0.0316}} = \frac{1.0079}{0.0316}$$

$$e^{0.0581t} = 31.8956$$

$$\frac{\ln(31.8956)}{0.0581} = \frac{\cancel{0.0581}t}{\cancel{0.0581}}$$

$$\boxed{t = 59.6 \text{ yrs}}$$

(d) The numerator is reasonable since the max % of wood products is 100%.