pg. 468

6.7 Financial Models

Compound Interest Formula:

$$A = P\cdot\left(1+\frac{r}{n}\right)^{(n\cdot t)}$$

A = ending amount
P = principal amount
r = rate (must be a decimal)
n = number of times compounded
t = time in yrs.

ex. 2) invest $1000 at an annual rate of 10% compounded _____ after 1 year.

(a) compounded semiannually:

$A = 1000\left(1+\frac{.10}{2}\right)^{(2\cdot 1)}$
$= \boxed{\$1102.50}$

P = 1000
r = .10
n = 2
t = 1

(b) compounded monthly:

$A = 1000\left(1+\frac{.10}{12}\right)^{(12\cdot 1)}$
$= \boxed{\$1104.71}$

P = 1000
r = .10
n = 12
t = 1

Continuous Compounding Formula:

$$A = Pe^{r\cdot t}$$

A = ending amount
P = principal amount
e = # e (2.71828)
r = rate
t = time in yrs.

ex. 3) invest $1000 at a rate of 10% compounded continuously for a time of 1 yr.

P = 1000
r = .10
t = 1

$A = 1000e^{(.10\cdot 1)} = \boxed{\$1105.17}$

Effective Rate of Interest formula:

compounding n times per yr: $r_e = \left(1 + \frac{r}{n}\right)^n - 1$

continuous compounding: $r_e = e^r - 1$

ex.4) You buy a 5yr CD. You visit 3 banks to determine their rates.

American Express: 2.15% annual int. comp. monthly
First Bank: 2.20% compounded quarterly
Discover: 2.12% compounded daily

which bank has the best deal?

Best deal = highest effective interest rate

American Express	First Bank	Discover
$r_e = \left(1 + \frac{0.0215}{12}\right)^{12} - 1$	$r_e = \left(1 + \frac{0.022}{4}\right)^4 - 1$	$r_e = \left(1 + \frac{0.0212}{365}\right)^{365} - 1$
$\approx 1.02171 - 1$	$\approx 1.02218 - 1$	$\approx 1.02143 - 1$
$= 0.02171$	$= 0.02218$	$= 0.02143$
$= 2.171\%$	$= 2.218\%$	$= 2.143\%$

↑ highest, so First Bank is best deal

Present Value Formula:

$$P = A \cdot \left(1 + \frac{r}{n}\right)^{(-nt)} \qquad P = Ae^{-rt}$$

(How much will you need to invest to recieve A dollars after t yrs)

ex.5) A zero-coupon bond can be redeemed in 10 yrs for $1000. How much should you be willing to pay for it now if you want a return of 8% compounded monthly?

$A = 1000$
$n = 12$
$r = 0.08$
$t = 10$

$$P = 1000 \cdot \left(1 + \frac{0.08}{12}\right)^{(-12 \cdot 10)} = \$450.52$$

ex.6) What interest rate (compounded annually) is needed in order to double an investment in 5 yrs?

$A = 2P$
$n = 1$
$t = 5$

$$2P = P \cdot \left(1 + \frac{r}{1}\right)^{(5 \cdot 1)}$$

$$\frac{2P}{P} = \frac{P(1+r)^5}{P}$$

$$\sqrt[5]{2} = \sqrt[5]{(1+r)^5}$$

$$\sqrt[5]{2} = 1 + r$$

$$-1 \quad -1$$

$$r = \sqrt[5]{2} - 1 \approx 1.148698 - 1 = 0.148698$$

rate would need to be 14.87%

ex.7) How long will it take to triple an investment if it earns 5% compounded continuously?

$A = 3P$
$r = 0.05$

e^{rt}

$$\frac{3P}{P} = \frac{Pe^{0.05t}}{P}$$

$$3 = e^{0.05t}$$

$$\frac{\ln(3)}{0.05} = \frac{0.05t}{0.05}$$

$$\rightarrow t = \frac{\ln(3)}{0.05} \approx 21.97$$

It will take about 22 yrs