

6.6 Log & Exponential Equations

* $y = \log_a X \iff X = a^y$

* If $\log_a M = \log_a N$ then $M = N$

ex. 1) solve: $2 \log_5 X = \log_5 9$

* Domain!! $x > 0$

$$\log_5 X^2 = \log_5 9$$

$$X^2 = 9$$

$X = 3$ or
 $X = -3$

← extraneous

ex. 2) $\log_5(x+6) + \log_5(x+2) = 1$

$$\log_5[(x+6)(x+2)] = 1 \quad \leftarrow \text{change to exp.}$$

$$5^1 = (x+6)(x+2)$$

$$5 = x^2 + 8x + 12$$

$$0 = x^2 + 8x + 7$$

$$0 = (x+1)(x+7)$$

$x = -1$

$x = -7$

← extraneous

* Domain!!

$x > -6$ and $x > -2$

so $x > -2$

ex. 3) $\ln x = \ln(x+6) - \ln(x-4)$

$$\ln x = \ln\left(\frac{x+6}{x-4}\right)$$

* Domain!!

$$(x-4)x = \frac{x+6}{x-4} \cdot (x-4)$$

$x > -6$ & $x > 4$

so $x > 4$

$$x^2 - 4x = x + 6$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$x = 6$, $x = -1$

←

ex. 4) solve: (a) $2^x = 5$
 $\log_2 5 = x$

$$x = \frac{\log 5}{\log 2} \approx \boxed{2.322}$$

(b) $8 \cdot 3^x = 5$

$$3^x = \frac{5}{8}$$

$$\log_3 \frac{5}{8} = x$$

$$\frac{\log \frac{5}{8}}{\log 3} \approx \boxed{-0.428}$$

ex. 9) solve: $5^{x-2} = 3^{3x+2}$

$$\ln 5^{x-2} = \ln 3^{3x+2}$$

$$(x-2) \ln 5 = (3x+2) \ln 3$$

*we want all x's on one side.

$$(\ln 5)x - 2 \ln 5 = (\ln 3)(3x) + 2 \ln 3$$

$$(\ln 5)x - (\ln 3)(3x) = 2 \ln 3 + 2 \ln 5$$

$$(\ln 5)x - (3 \ln 3)(x) = 2 \ln 3 + 2 \ln 5$$

$$x \frac{(\ln 5 - 3 \ln 3)}{\ln 5 - 3 \ln 3} = \frac{2(\ln 3 + \ln 5)}{\ln 5 - 3 \ln 3}$$

$$x = \frac{2(\ln 3 + \ln 5)}{\ln 5 - 3 \ln 3} \approx \boxed{-3.212}$$

ex. 10) solve: $4^x - 2^x - 12 = 0$

$$\uparrow$$

$$(2^2)^x \text{ or } (2^x)^2$$

think of $2^x = u$
 so $u^2 - u - 12$

so $(2^x)^2 - 2^x - 12 = 0$

$$(2^x - 4)(2^x + 3) = 0$$

$$2^x - 4 = 0 \quad 2^x + 3 = 0$$

$$2^x = 4$$

$$\boxed{x=2}$$

$$2^x = -3$$

NO SOLUTION