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6.5 Properties of Logs

* **Properties:** ① $\log_a 1 = 0$ $\quad \log_a a = 1$

② $a^{\log_a M} = M$

③ $\log_a a^r = r$

④ $\log_a(MN) = \log_a M + \log_a N$

⑤ $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

⑥ $\log_a M^r = r \log_a M$

⑦ $a^r = e^{r \ln a}$

ex. 3) $\log_a(x\sqrt{x^2+1}), x>0$

$$\log_a(x) + \log_a(\sqrt{x^2+1}) = \log_a(x) + \log_a(x^2+1)^{1/2}$$

$$= \boxed{\log_a(x) + \tfrac{1}{2}\log_a(x^2+1)}$$

ex. 4) $\ln \frac{x^2}{(x-1)^3}$, $x>1$

$$\ln x^2 - \ln(x-1)^3 = \boxed{2\ln x - 3\ln(x-1)}$$

ex. 5) $\log_a \frac{\sqrt{x^2+1}}{x^3(x+1)^4}$, $x>0$

$$\log_a \sqrt{x^2+1} - \log_a[x^3(x+1)^4]$$

$$\underbrace{\log_a \sqrt{x^2+1}}_{\log_a (x^2+1)^{1/2}} - [\log_a x^3 + \log_a (x+1)^4]$$

$$= \boxed{\tfrac{1}{2}\log_a(x^2+1) - 3\log_a x - 4\log_a(x+1)}$$

ex.6) write as single log:

(a) $\log_a 7 + 4\log_a 3 = \log_a 7 + \log_a 3^4$

$= \log_a 7 + \log_a 81$

$= \log_a (7 \cdot 81) = \boxed{\log_a 567}$

(b) $\frac{2}{3}\ln 8 - \ln(5^2 - 1) = \ln 8^{2/3} - \ln(25-1)$

$= \ln 4 - \ln(24)$

$= \ln\left(\frac{4}{24}\right) = \ln\left(\frac{1}{6}\right) = \ln 1 - \ln 6$

$= \boxed{-\ln 6}$

(c) $\log_a x + \log_a 9 + \log_a(x^2+1) - \log_a 5$

$= \log_a(9x) + \log_a(x^2+1) - \log_a 5$

$= \log_a[9x(x^2+1)] - \log_a 5$

$= \boxed{\log_a\left[\frac{9x(x^2+1)}{5}\right]}$

* Change-of-Base Formula:

$$\log_a M = \frac{\log_b M}{\log_b a}$$

ex.9) Approximate:

(a) $\log_5 89$

$= \frac{\log 89}{\log 5} \approx \frac{1.949}{0.698}$

$\approx \boxed{2.7889}$

(b) $\log_{\sqrt{2}}\sqrt{5} = \frac{\log\sqrt{5}}{\log\sqrt{2}} \approx \frac{0.4886}{0.1814}$

$\approx \boxed{2.7889}$