6.4 Logarithmic Functions

$y = \log_a x$ if and only if $x = a^y$

ex.1) (a) $y = \log_3 x \leftrightarrow x = 3^y$ or

$4 = \log_3 81 \leftrightarrow 3^4 = 81$

ex.2) (a) $1.2^3 = m$ (b) $e^b = 9$ (c) $a^4 = 24$

$\log_{1.2}(m) = 3$ $\log_e(9) = b$ $\log_a(24) = 4$

ex.3) (a) $\log_a 4 = 5$ (b) $\log_e b = -3$ (c) $\log_3 5 = c$

$a^5 = 4$ $e^{-3} = b$ $3^c = 5$

ex.4) (a) Find $\log_2 16$

$y = \log_2 16$

$2^y = 16$

or $2^y = 2^4$

$\boxed{y = 4}$

(b) $\log_3 \frac{1}{27}$

$y = \log_3 \frac{1}{27}$

$3^y = \frac{1}{27}$

$3^y = 3^{-3}$

$\boxed{y = -3}$

*Domain of a log: anything in parenthesis has to be greater than 0

ex.5) (a) $F(x) = \log_2(x+3) \leftarrow x+3 > 0$ or $x > -3$ $\boxed{(-3, +\infty)}$

(b) $g(x) = \log_5\left(\frac{1+x}{1-x}\right) \leftarrow \frac{1+x}{1-x} > 0$ true for $-1 < x < 1$ or $\boxed{(-1, 1)}$

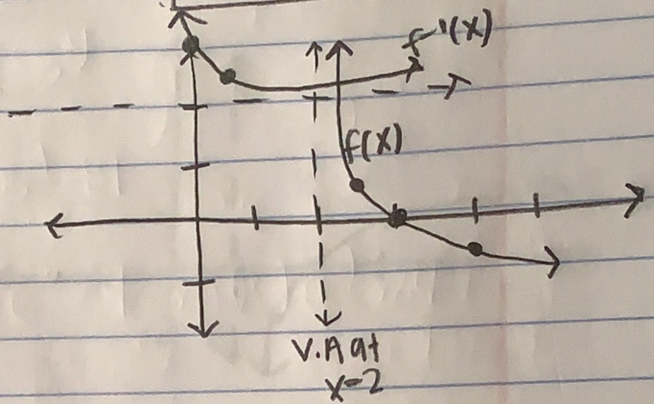
(c) $h(x) = \log_{1/2}|x| \leftarrow \boxed{(-\infty, 0) \cup (0, \infty)}$

ex.6) $f(x) = -\ln(x-2)$

(a) domain $x-2>0 \rightarrow x>2$ or $\boxed{(2,+\infty)}$

(b) graph

x	y
2.5	0.69
3	0
4	-0.69



(c) range: $(-\infty, +\infty)$

V.A.: $x=2$

(d) inverse of f(x)?

$y = -\ln(x-2)$

$x = -\ln(y-2)$

$-x = \ln(y-2)$

$e^{-x} = y-2$

$\boxed{f^{-1}(x) = e^{-x}+2 = y}$

(e) domain of f^{-1}: $(-\infty, +\infty)$

range of f^{-1}: $(2, +\infty)$

(f) graph of f^{-1}:

x	y
0.69	2.5
0	3
-0.69	4

log with no base = base 10

$y = \log x$

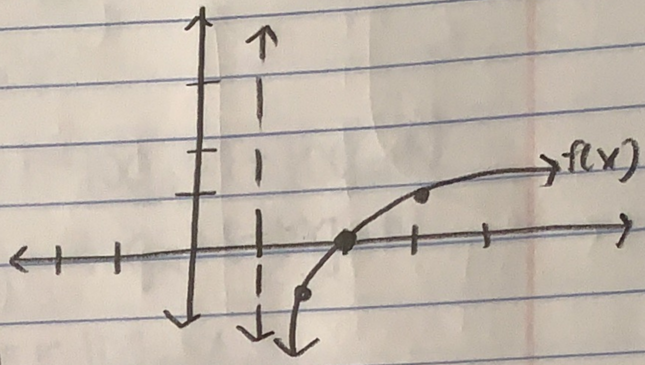
↑ base 10

ex.7) $f(x) = 3\log(x-1)$

(a) domain: $x-1>0 \rightarrow x>1$ or $(1,+\infty)$

(b) graph:

x	y
1.5	-0.9
2	0
3	0.9



(c) range: $(-\infty,+\infty)$

V.A.: $x=1$

ex.8) solve:

(a) $\log_3(4x-7)=2$

$3^2=4x-7$

$9=4x-7$

$16=4x$

$\boxed{x=4}$

(b) $\log_x 64=2$

$x^2=64$

$x=\pm\sqrt{64}$

$\boxed{x=\pm 8}$

ex.9) solve: $e^{2x}=5$

$\ln 5=2x$

$x=\frac{\ln 5}{2} \approx \boxed{0.805}$