pg. 423

# 6.3 Exponential Functions

**Law of Exponents**: if $s, t, a, \& b$ are real #s with $a>0$ and $b>0$ then,

$$a^s \cdot a^t = a^{s+t} \qquad (a^s)^t = a^{st} \qquad (ab)^s = a^s \cdot b^s$$

$$1^s = 1 \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \qquad a^0 = 1$$

**Exponential Form**: $f(x) = Ca^x$ ($C \neq 0$, $a > 0$, $a \neq 1$)

$C$ = initial value
$a$ = growth factor

ex.2) Linear or exponential?

(a)

| x | y | |
|---|---|---|
| -1 | 5 | |
| 0 | 2 | -3 |
| 1 | -1 | -3 |
| 2 | -4 | -3 |
| 3 | -7 | -3 |

since -3 everytime the table represents a LINEAR function

(b)

| x | y | |
|---|---|---|
| -1 | 32 | ÷2 |
| 0 | 16 | ÷2 |
| 1 | 8 | ÷2 |
| 2 | 4 | ÷2 |
| 3 | 2 | |

since ÷2 this table represents an EXPONENTIAL

(c)

| x | y | |
|---|---|---|
| -1 | 2 | |
| 0 | 4 | +2 |
| 1 | 7 | +3 |
| 2 | 11 | +4 |
| 3 | 16 | +5 |

not constant

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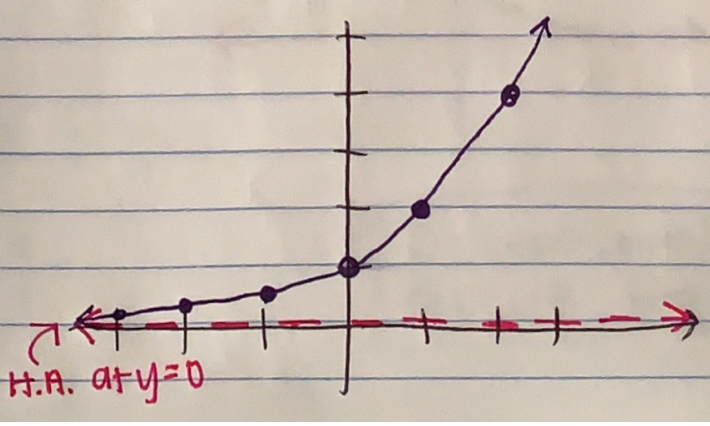
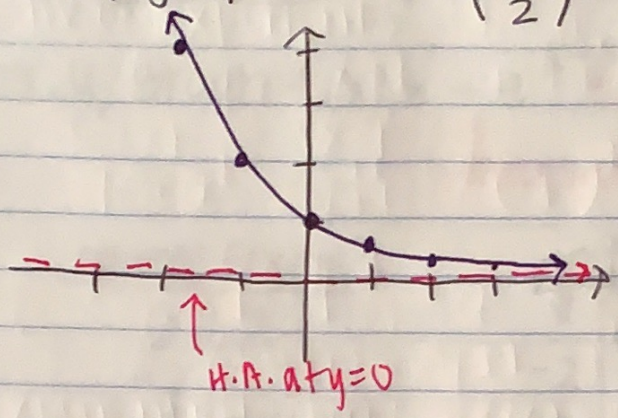
ex.3) Graph $f(x) = 2^x$




| x | y |
|---|---|
| -3 | 1/8 |
| -2 | 1/4 |
| -1 | 1/2 |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |

ex.4) graph $f(x) = \left(\frac{1}{2}\right)^x$




| x | y |
|---|---|
| -3 | 8 |
| -2 | 4 |
| -1 | 2 |
| 0 | 1 |
| 1 | 1/2 |
| 2 | 1/4 |
| 3 | 1/8 |

ex. 5) Graph using transformations: $f(x) = 2^{-x} - 3$ ← down 3

↑ reflect over y

parent points: $2^x$

| x | y |
|---|---|
| -2 | 1/4 |
| -1 | 1/2 |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |

$2^{-x}$

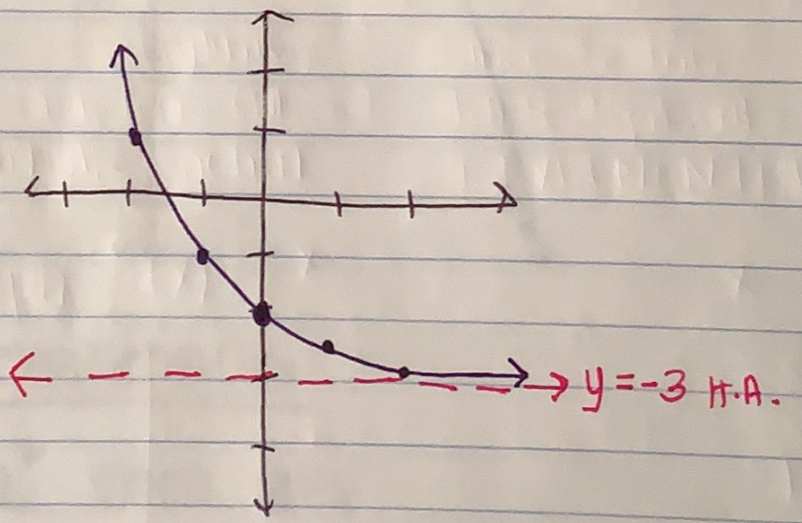
| x | y |
|---|---|
| 2 | 1/4 |
| 1 | 1/2 |
| 0 | 1 |
| -1 | 2 |
| -2 | 4 |

↑ change signs

$2^{-x} - 3$

| x | y |
|---|---|
| 2 | -2.75 |
| 1 | -2.5 |
| 0 | -2 |
| -1 | -1 |
| -2 | 1 |

↑ -3 from y




D: $(-\infty, +\infty)$

R: $(-3, +\infty)$

* The number e : 2.71828|828459.....

ex. a) Graph $f(x) = -e^{x-3}$ ↑ reflect over x, ↑ Right 3

parent: $f(x) = e^x$

| x | y |
|---|---|
| -2 | 0.135 |
| -1 | 0.368 |
| 0 | 1 |
| 1 | 2.718 |
| 2 | 7.4 |

$-e^x$

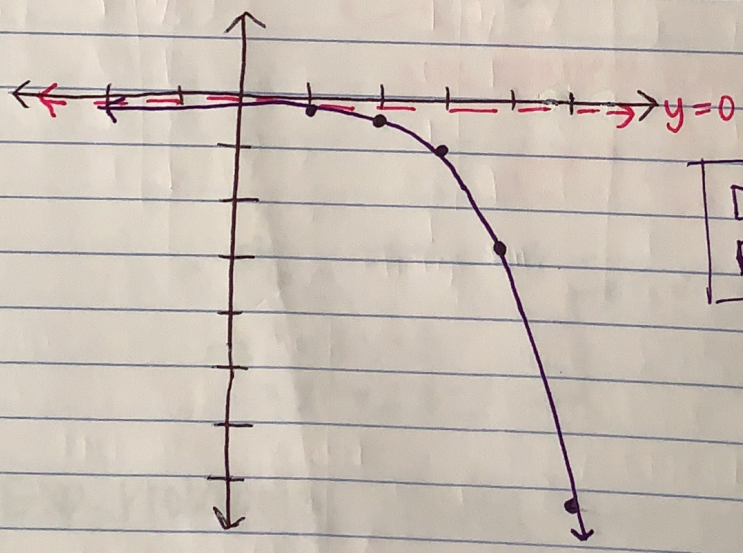
| x | y |
|---|---|
| -2 | -0.14 |
| -1 | -0.37 |
| 0 | -1 |
| 1 | -2.72 |
| 2 | -7.4 |

↑ y signs

$-e^{x-3}$

| x | y |
|---|---|
| 1 | -0.14 |
| 2 | -0.37 |
| 3 | -1 |
| 4 | -2.72 |
| 5 | -7.4 |

↑ add 3




D: $(-\infty, +\infty)$
R: $(-\infty, 0)$

## Solving exponentials

* If $a^u = a^v$ then $u = v$.

ex. $4^2 = 4^{x+3}$ then $2 = x+3$ so $x = -1$

ex.7) solve:

a) $3^{x+1} = 81$

need to make a base 3

$3^{x+1} = 3^4$

$x+1 = 4$

$\boxed{x = 3}$

b) $4^{2x-1} = 8^{x+3}$

change both bases to 2

$(2^2)^{2x-1} = (2^3)^{x+3}$

$2^{2(2x-1)} = 2^{3(x+3)}$

$2(2x-1) = 3(x+3)$

$4x - 2 = 3x + 9$

$-3x + 2 \quad -3x + 2$

$\boxed{x = 11}$

ex.8) solve: $e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$

use exponent rules

$(e^x)^2 \cdot \frac{1}{e^3} = e^{2x} \cdot e^{-3} = e^{2x-3}$

$e^{-x^2} = e^{2x-3}$

$-x^2 = 2x - 3$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$\boxed{x = -3 \text{ or } x = 1}$

ex.9) Exp. Probability : given $F(t) = 1 - e^{-0.2t}$

(a) prob. car will arrive within 5 min:

$F(5) = 1 - e^{-0.2(5)} = 0.63212 \rightarrow \boxed{63\%}$

(b) prob. car will arrive within 30 min:

$F(30) = 1 - e^{-0.2(30)} = 0.9975 \rightarrow \boxed{99.8\%}$

HW: pg. 434 #15-39 odd, 95-99 odd