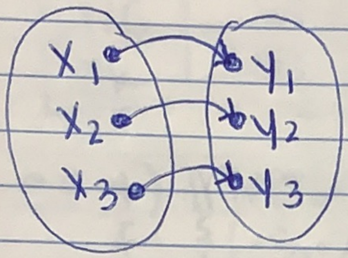
pg. 411

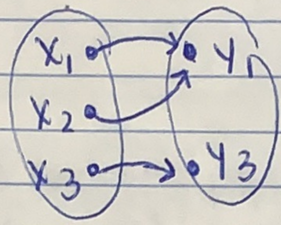
6.2 one-to-one ; inverse Functions

* one-to-one ← has to be a function and
2 inputs cannot correspond
to same output.

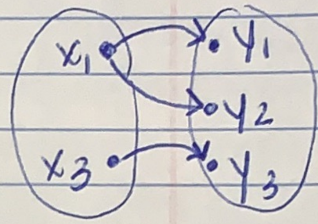
one-to-one



Function But Not one-to-one



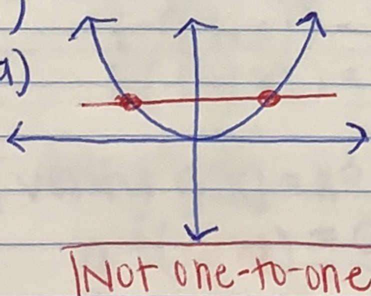
Not one-to-one Not a function



ex.1) Look at book pg. 412 for figure
a) not one-to-one (2 different inputs have same out)
b) is one-to-one.

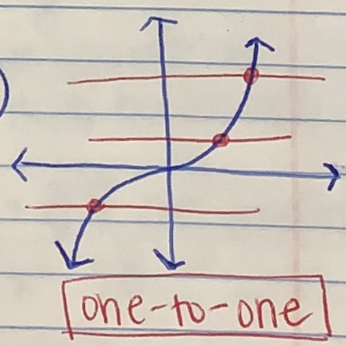
* Horizontal line test - if a horizontal line intersects
graph at only one point, it is one-to-one

ex.2)
a)



Not one-to-one

b)



one-to-one

inverse functions

* The symbol f^{-1} is used to denote the
inverse function of f

ex.3) inverse of a map

state		pop (in millions)
IN	→	6.5
WA	→	6.9
SD	→	0.8
NC	→	9.8
OK	→	3.8

inverse:

pop (in millions)		state
6.5	→	IN
6.9	→	WA
0.8	→	SD
9.8	→	NC
3.8	→	OK

ex.4) inverse of ordered pair

$f: \{(-3,-27)(-2,-8)(-1,-1)(0,0)(1,1)(2,8)(3,27)\}$

inverse:

$f^{-1}: \{(-27,-3)(-8,-2)(-1,-1)(0,0)(1,1)(8,2)(27,3)\}$

Domain of f & Range of f^{-1}:

$\{-3,-2,-1,0,1,2,3\}$

Range of f & Domain of f^{-1}:

$\{-27,-8,-1,0,1,8,27\}$

ex.5) verify inverse : use:

$f^{-1}(f(x)) = x$ where x is in the domain of f

$f(f^{-1}(x)) = x$ where x is in the domain of $f^{-1}(x)$.

a) verify $g(x) = x^3$ and $g^{-1}(x) = \sqrt[3]{x}$ are inverses:

$g^{-1}(g(x)) = g^{-1}(x^3) = \sqrt[3]{x^3} = x$ ✓

$g(g^{-1}(x)) = g(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$ ✓

→

ex.6) verify the inverse of $f(x)=\frac{1}{x-1}$ & $f^{-1}(x)=\frac{1}{x}+1$

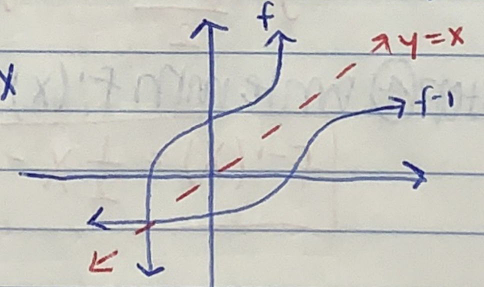
$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x-1}\right) = \frac{1}{\frac{1}{x-1}}+1 = x-1+1 = x$$ $\boxed{\text{only if } x \neq 1}$

$$f(f^{-1}(x)) = f\left(\frac{1}{x}+1\right) = \frac{1}{\left(\frac{1}{x}+1\right)-1} = \frac{1}{\frac{1}{x}} = x$$ $\boxed{\text{only if } x \neq 0}$

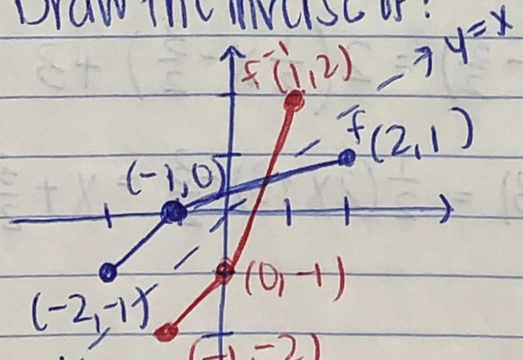
ex.7) Graphing inverse

* the graph of a one-to-one function f and the graph of its inverse function f^{-1} are symmetric over the line $y=x$.

ex

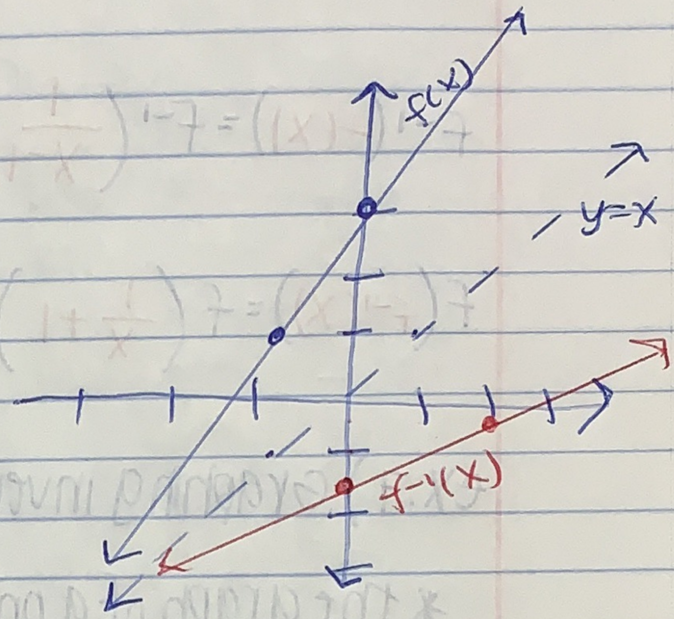


Draw the inverse of:



$f(x)$		$f^{-1}(x)$
(-2,-1)	→	(-1,-2)
(-1,0)	→	(0,-1)
(2,1)	→	(1,2)

ex. 8) Find inverse function for $f(x) = 2x+3$
& graph them.



step ① replace f(x) with y

$y = 2x+3$

step ② switch the x & y

$x = 2y+3$

step ③ solve for y

$\frac{x-3}{2} = \frac{2y}{2}$

$y = \frac{x-3}{2}$ or $y = \frac{1}{2}x - \frac{3}{2}$

step ④ write with $f^{-1}(x)$

$\boxed{f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}}$

check answer:

$f(f^{-1}(x)) = f(\frac{1}{2}x - \frac{3}{2}) = 2(\frac{1}{2}x - \frac{3}{2}) + 3 = (x-3)+3 = x$ ✓

$f^{-1}(f(x)) = f^{-1}(2x+3) = \frac{1}{2}(2x+3) - \frac{3}{2} = x + \frac{3}{2} - \frac{3}{2} = x$ ✓

ex.9) Find inverse of $f(x) = \frac{2x+1}{x-1}$ $x \neq 1$

① $y = \frac{2x+1}{x-1}$

② $x = \frac{2y+1}{y-1}$

③ $x(y-1) = 2y+1$

$xy - x = 2y + 1$ ← get y's on one side

$xy - 2y = x + 1$

$y(x-2) = x+1$

$y = \frac{x+1}{x-2}$

④ $\boxed{f^{-1}(x) = \frac{x+1}{x-2} \quad x \neq 2}$