

6.1 composite functions

$$(f \circ g)(x) = f(g(x))$$

↑
read "f composed with g"

ex.1) given $f(x) = 2x^2 - 3$ $g(x) = 4x$ find:

$$a) (f \circ g)(1) = f(g(1)) = f(4) = 2(4)^2 - 3 = \boxed{29}$$

↑
 $g(1) = 4(1)$
 $g(1) = 4$

$$b) (g \circ f)(1) = g(f(1)) = g(-1) = 4(-1) = \boxed{-4}$$

↑
 $f(1) = 2(1)^2 - 3 = -1$

$$c) (f \circ f)(-2) = f(f(-2)) = f(5) = 2(5)^2 - 3 = \boxed{47}$$

↑
 $f(-2) = 2(-2)^2 - 3 = 5$

$$d) (g \circ g)(-1) = g(g(-1)) = g(-4) = 4(-4) = \boxed{-16}$$

ex.2) $f(x) = x^2 + 3x - 1$ and $g(x) = 2x + 3$

Find a) $f \circ g$ and b) $g \circ f$ and their domain:

$$a) f \circ g = f(g(x)) = (2x+3)^2 + 3(2x+3) - 1$$

$$= 4x^2 + 12x + 9 + 6x + 9 - 1 = \boxed{4x^2 + 18x + 17}$$

$$b) g \circ f = g(f(x)) = 2(x^2 + 3x - 1) + 3$$

$$= 2x^2 + 6x - 2 + 3 = \boxed{2x^2 + 6x + 1}$$

* since domain of f & g are \mathbb{R} , the domain for $f \circ g$ & $g \circ f$ are \mathbb{R} .

ex.3) Find domain of $f \circ g$ if $f(x) = \frac{1}{x+2}$ & $g(x) = \frac{4}{x-1}$

$$f(g(x))$$

↑
 $x \neq 1$

$g(x) \neq -2$
what x value is this me??

$$\frac{4}{x-1} = -2$$

$$4 = -2(x-1)$$

$$4 = -2x + 2$$

$$2 = -2x$$

$$\rightarrow x = -1$$

$$x+2$$

$$\uparrow$$

$$x \neq -2$$

$$x-1$$

$$\uparrow$$

$$x \neq 1$$

so $D: \{x \mid x \neq -1, 1\}$

ex. 4) $f(x) = \frac{1}{x+2}$ $g(x) = \frac{4}{x-1}$

a) find $f \circ g$: $f(g(x)) = \frac{1}{\left(\frac{4}{x-1}\right) + \frac{2(x-1)}{x-1}} = \frac{1}{\frac{4}{x-1} + \frac{2x-2}{x-1}} = \frac{1}{\frac{2x+2}{x-1}}$

$f(g(x)) = \frac{x-1}{2x+2}$ $D: \{x \mid x \neq -1, 1\}$

b) find $f \circ f$: $f(f(x)) = f\left(\frac{1}{x+2}\right) = \frac{1}{\left(\frac{1}{x+2}\right) + \frac{2(x+2)}{x+2}}$

$= \frac{1}{\frac{1}{x+2} + \frac{2x+4}{x+2}} = \frac{1}{\frac{2x+5}{x+2}} = \frac{x+2}{2x+5}$

domain: $\{x \mid x \neq -2, -\frac{5}{2}\}$

ex. 5) If $f(x) = 3x - 4$ & $g(x) = \frac{1}{3}(x + 4)$

show $(f \circ g)(x) = (g \circ f)(x) = x$

$(f \circ g)(x)$	$(g \circ f)(x)$
$f(g(x)) = f\left(\frac{x+4}{3}\right)$	$g(f(x)) = g(3x-4)$
$= 3\left(\frac{x+4}{3}\right) - 4$	$= \frac{1}{3}[(3x-4) + 4]$
$= x + 4 - 4$	$= \frac{1}{3}[3x]$
$= x \quad \checkmark$	$= \frac{1}{3}(3x) = \frac{3x}{3} = x \quad \checkmark$

so $(f \circ g)(x) = (g \circ f)(x) = x$

ex. 6) Find f and g such that $f \circ g = H$ if

$$H(x) = \underbrace{(x^2+1)}_{x^2+1} \overset{\text{raised to } 50}{50}$$

let $\boxed{f(x) = x^{50} \text{ \& } g(x) = x^2+1}$ so

$$f(g(x)) = f(x^2+1) = (x^2+1)^{50} = H(x) \checkmark$$

ex. 7) Find f & g such that $f \circ g = H$ if $H(x) = \frac{1}{x+1}$ $\left. \begin{array}{l} \text{?} \\ \downarrow \\ \frac{1}{x} \end{array} \right\}$

let $\boxed{f(x) = \frac{1}{x} \text{ \& } g(x) = x+1}$

$$\text{so } f(g(x)) = f(x+1) = \frac{1}{x+1} = H(x) \checkmark$$

HW: pg. 408-409 # 9-47 odd