5.6 Complex Zeros

* **Fundamental Theorem of Algebra** - Every complex poly function f of degree $n \geq 1$ has at least one complex zero.

* **conjugate pairs theorem** - Let f be a poly whose coefficients are real numbers. If $r = a+bi$ is a zero of f, the complex conjugate $\bar{r} = a-bi$ is a zero of f.

ex.1) A poly f of degree 5 whose coefficients are real #s has the zeros $1, 5i$, and $1+i$. Find the remaining two zeros.

since $x = 5i$ is a zero $\rightarrow$ $\boxed{x = -5i}$

since $x = 1+i$ is a zero $\rightarrow$ $\boxed{x = 1-i}$

ex.2) Find the poly function of f of degree 4 whose zeros are $1, 1$, and $-4+i$.

$x = 1 \quad x = 1 \quad x = -4+i \quad \text{\&} \quad x = -4-i$

$-1 \quad -1 \quad -1 \quad -1 \quad +4-i$

$x-1=0 \quad x-1=0 \quad x+4-i=0 \quad x+4+i=0$

$f(x) = a(x-1)(x-1)(x+4-i)(x+4+i)$

$f(x) = a(x^2-2x+1)(x^2+8x+16-i^2)$

$= a(x^2-2x+1)(x^2+8x+17)$

$\boxed{f(x) = a(x^4+6x^3+2x^2-26x+17)}$

	x	4	$-i$
x	x^2	$4x$	$-ix$
4	$4x$	16	$-4i$
i	ix	$+4i$	$-i^2$

	x^2	$-2x$	1
x^2	x^4	$-2x^3$	x^2
$8x$	$8x^3$	$-16x^2$	$8x$
17	$17x^2$	$-34x$	17

ex.3) find complex zeros of $f(x)=3x^4+5x^3+25x^2+45x-18$

at most 4 zeros

- 1 + real zero
- $f(-x) = 3x^4-5x^3+25x^2-45x-18$

3 or 1 - real zero

- possible rational zeros:

$p: \pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

$q: \pm1, \pm3$

so $\frac{p}{q}: \pm\frac{1}{3}, \pm\frac{2}{3}, \pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

- check: $f(1) = 60$ $\quad f(-1) = -40$ $\quad$ etc...

$f(2) = 260$ $\quad f(-2) = 0$

- divide:

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & 25 & 45 & -18 \\ & \downarrow & -6 & 2 & -54 & 18 \\ \hline & 3 & -1 & 27 & -9 & 0 \end{array}$$

$$(x+2)\underbrace{(3x^3-1x^2+27x-9)}_{\text{factor}}$$

$$\begin{array}{c|c|c} & 3x & -1 \\ \hline x^2 & 3x^3 & -1x^2 \\ \hline 9 & 27x & -9 \end{array}$$

$$(x+2)(x^2+9)(3x-1)$$

$$\boxed{x=-2, -3i, +3i, \frac{1}{3}}$$