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5.4 Polynomial & Rational Inequalities

ex.2) Solve a poly inequality Algebraically

Solve $x^4 > +1$

① Put everything on one side:

$$x^4 - 1 > 0$$

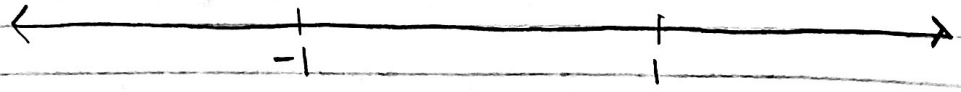
② Find real zeros:

$$x^4 - 1 = 0$$
$$(x^2+1)(x^2-1) = 0$$
$$(x^2+1)(x+1)(x-1) = 0$$
$$x = 1 \text{ or } x = -1 \text{ or } \underbrace{x^2+1=0}_{\text{non-real}}$$

③ Use real zeros to seperate the #line into 3 intervals.

$$(-\infty, -1) \quad (-1, 1) \quad (1, +\infty)$$

④ Find a test # in each interval & evaluate it in $f(x) = x^4 - 1$ to determine if it is positive or negative.



interval:	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
# Chosen:	-2	0	2
value of f:	$f(-2) = (-2)^4 - 1 = 15$	$f(0) = (0)^4 - 1 = -1$	$f(2) = (2)^4 - 1 = 15$
conclusion:	positive	neg	pos

*positive means > 0
*negative means < 0

So $x^4 - 1 > 0$ from $(-\infty, -1) \cup (1, +\infty)$
or $\{x \mid x < -1 \text{ or } x > 1\}$

ex4) Solve a rational inequality algebraically.

solve $\frac{3x^2+13x+9}{(x+2)^2} \leq 3$

① make 0 on the right side

$$\frac{3x^2+13x+9}{x^2+4x+4} - 3 \leq 0$$

subtract

$$\frac{3x^2+13x+9}{x^2+4x+4} - \frac{3(x^2+4x+4)}{x^2+4x+4} \leq 0$$

$$\frac{3x^2+13x+9-(3x^2+12x+12)}{x^2+4x+4} \leq 0$$

$$\frac{x-3}{x^2+4x+4} \leq 0$$

② identify zeros & #s for which x is undefined

$f(x) = \frac{x-3}{x^2+4x+4}$ $\quad 0 = \frac{x-3}{x^2+4x+4}$ $\quad 0 = x-3$ so $x = 3$

$x^2+4x+4 = 0$ also $x \neq -2$

$(x+2)(x+2) = 0$

$x = -2$

③ # test:

$f(-3) = -6$ $\quad f(0) = -\frac{3}{4}$ $\quad f(4) = \frac{1}{36}$

so $\frac{x-3}{x^2+4x+4} \leq 0$ at $(-\infty, -2) \cup (-2, 3]$

or $\{x \mid x < -2 \text{ or } -2 < x \leq 3\}$