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5.2 properties of Rationals

* Rational is in the form $R(x) = \frac{p(x)}{q(x)}$ where p & q are polynomials & q is not zero.

* **Domain of a Rational**

(a) $\frac{2x^2-4}{x+5}$ $\boxed{D: \{x \mid x \neq -5\}}$

(b) $R(x) = \frac{1}{x^2-4} = \frac{1}{(x+2)(x-2)}$ $\boxed{D: \{x \mid x \neq -2, 2\}}$

(c) $R(x) = \frac{x^3}{x^2+1}$ $\boxed{D: \mathbb{R}}$

(d) $R(x) = \frac{x^2-1}{x-1}$ $\boxed{D: \{x \mid x \neq 1\}}$

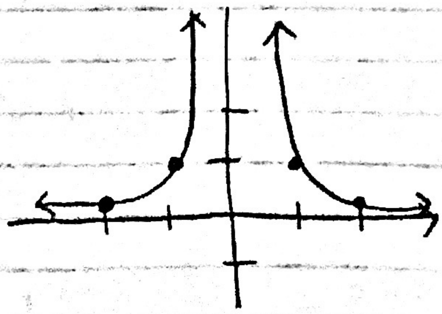
* Graphing $y = \frac{1}{x^2}$

x-int: $0 = \frac{1}{x^2} \rightarrow 0 \neq 1 \rightarrow$ no sol.

y-int: $y = \frac{1}{0^2} \rightarrow$ no sol.

graph doesnt touch x or y-axis

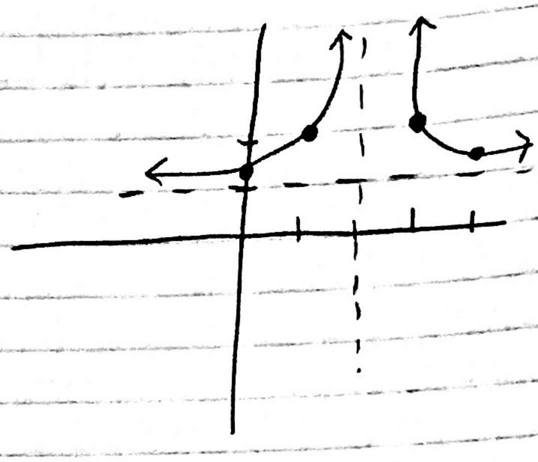
X	Y
−2	1/4
−1	1
1	1
2	1/4



* Transformation w/ Rationals

Graph $R(X) = \frac{1}{(X-2)^2} + 1$ ← up 1

↑ Right 2



$(-2, 1/4) \to (0, 1/4) \to (0, 1\tfrac{1}{4})$
$(-1, 1) \to (1, 1) \to (1, 2)$
$(1, 1) \to (3, 1) \to (3, 2)$
$(2, 1/4) \to (4, 1/4) \to (4, 1\tfrac{1}{4})$

* Asymptotes

Locating Vertical Asymptotes:

occurs at $X = r$ if $R(X) = \frac{p(X)}{q(X)}$ is simplified

and $x-r$ is a factor of the denominator q.

ex) Find the V.A.'s

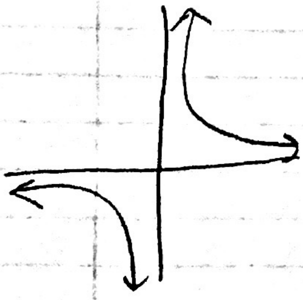
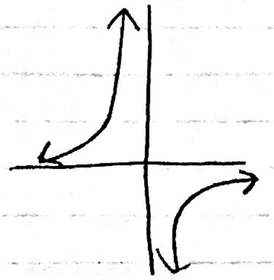
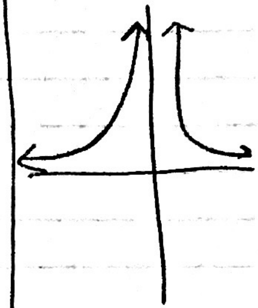
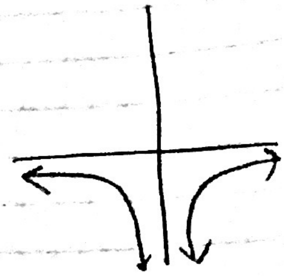
(a) $F(X) = \frac{X+3}{X-1}$ V.A. at $X = 1$

(b) $R(X) = \frac{X}{X^2-4}$ V.A. at $X = -2$ and $X = 2$

(c) $H(X) = \frac{X^2}{X^2+1}$ NO V.A.

(d) $G(X) = \frac{X^2-9}{X^2+4X-21} = \frac{(X+3)\cancel{(X-3)}}{(X+7)\cancel{(X-3)}} = \frac{X+3}{X+7}$ V.A at $X = -7$

Multiplicity w/ V.A.'s

odd multiplicity		even multiplicity	
$\frac{1}{x}$	$-\frac{1}{x}$	$\frac{1}{x^2}$	$-\frac{1}{x^2}$
			

Horizontal Asymptotes

given $R(x) = \frac{x^n}{x^m}$

*A rational function will never have a H.A. AND an oblique A.

- if $n < m$ then H.A. is $y = 0$
- if $n = m$ then H.A. is $y = \frac{a_n}{a_m}$
- if $n = m+1$ then there is an oblique asymptote (find by dividing)
- if $n \geq m+2$ no H.A. or oblique.

ex. 5) find the H.A.

$$R(x) = \frac{4x^3 - 5x}{7x^5 + 2x^4}$$

$\boxed{\text{since } 3<5 \text{ the H.A. is } y=0}$

ex. 6) find H.A. or oblique A of

$$H(x) = \frac{3x^4 - x^2}{x^3 - x^2 + 1}$$

since $4 > 3$ $H(x)$ has an oblique A.

$$H(x) = \frac{3x^4 + 0x^3 - x^2 + 0x + 0}{x^3 - x^2 + 0x + 1}$$

	$3x$	$+3$	
x^3	$3x^4$	$3x^3$	$2x^2$
$-x^2$	$-3x^3$	$-3x^2$	$-3x$
$+0x$	$0x^2$	$0x$	-3
$+1$	$3x$	3	

remainder

$$= 3x + 3 + \underbrace{\frac{2x^2 - 3x - 3}{x^3 - x^2 + 1}}$$

as $x \to +\infty$ or $-\infty$ this # will get $\to 0$

$\boxed{\text{so there is an oblique asymptote at } y = 3x + 3}$

ex. 7) find H.A. of $R(x) = \dfrac{8x^2 - x + 2}{4x^2 - 1}$

since $2 = 2$, you find ratio of leading coefficients

$$y = \frac{8}{4} = 2 \quad \boxed{y = 2}$$

ex. 8) Find H.A. or oblique

$$G(x) = \frac{2x^5 - x^3 + 2}{x^3 - 1}$$

$\boxed{\text{since } 5 \geq 3 + 2 \text{ there are no H.A. or oblique A.}}$