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5.1 Polynomial Functions & Models

* <u>Polynomial</u> - in one variable is a function in the form

$$f(x) = \underbrace{a_n x^n}_{\text{leading term}} + a_{n-1}x^{n-1} + \dots + a_1 x + \underbrace{a_0}_{\text{constant term}}$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are <u>constants</u>
if $n \geq 0$ then is called a <u>coefficients</u>
If $a_n \neq 0$ it is called the <u>leading coefficient</u>.
and n is the <u>degree</u> of the polynomial.

Polynomial or Not?

(a) $p(x) = 5x^3 - \frac{1}{4}x^2 - 9$ → Poly (degree 3, leading term $5x^3$, constant -9, in standard form)

(b) $f(x) = x + 2 - 3x^4$ → Poly (degree 4, leading term $-3x^4$, constant 2)
<u>in standard</u>: $-3x^4 + x + 2$

(c) $g(x) = \sqrt{x}$ → Not a poly

(d) $h(x) = \dfrac{x^2 - 2}{x^3 - 1}$ → Not a poly

(e) $G(x) = 8$ → Poly (called a nonzero constant poly, degree 0, leading term/constant is 8)

(f) $H(x) = -2x^3(x-1)^2$ → Poly (degree 5, leading term $-2x^5$, constant 0)
$-2x^3(x^2 - 2x + 1)$
$H(x) = -2x^5 + 4x^4 - 2x^3$

* Polynomials are smooth (no sharp corners) & continuous

* <u>Power Functions</u>: a monomial in the form $f(x) = ax^n$ where a is $\mathbb{R}$ and $a \neq 0$, and $n > 0$ & an integer

examples of power functions:

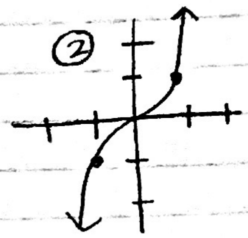
$f(x)=3x$ $\quad f(x)=-5x^2$ $\quad f(x)=8x^3$ $\quad f(x)=-5x^4$

ex.2) Graphing a poly using transformations

graph $f(x)=1-x^5$

1 Rewrite $f(x)=-x^5+1$

2 graph $y=x^5 \rightarrow (-1,-1)(0,0)(1,1)$

2


3 graph $y=-x^5$ (mult y-values by -1)

$(-1,1)(0,0)(1,-1)$

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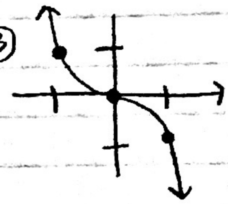
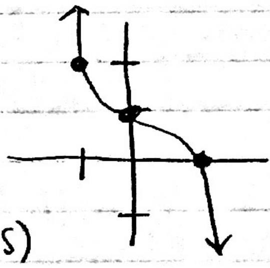


4 graph $y=-x^5+1$

$\uparrow$ up 1

(add 1 to the y values)

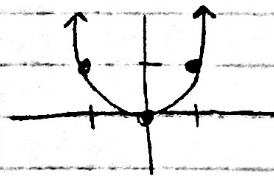
$(-1,2)(0,1)(1,0)$



ex.3) Graph $f(x)=\frac{1}{2}(x-1)^4$

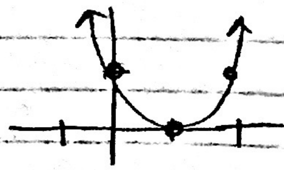
1 graph $y=x^4$

$(-1,1)(0,0)(1,1)$



2 graph $y=(x-1)^4$ (right 1)

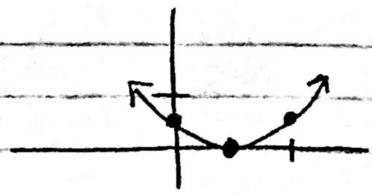
$\rightarrow$ add 1 to x-values $(0,1)\ (1,0)(2,1)$



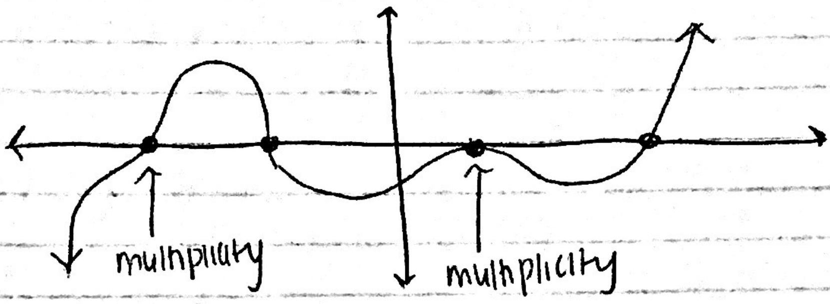
3 graph $y=\frac{1}{2}(x-1)^4$

(mult. y-values by $1/2$)

$(0,1/2)(1,0)(2,1/2)$



* Real zeros & their multiplicity



- if a function "f" and a real number "r" for which $f(r)=0$, then r is a real zero.

  which means... → r is an x-int of f
  x-r is a factor of f
  r is a solution to $f(x)=0$

- if $(x-r)^m$ is a factor of a poly then r is called a zero of multiplicity.

ex) $f(x)=\underbrace{(x+3)^2}_{\text{multiplicity at the zero } x=-3}\underbrace{(x-5)}_{\text{zero } x=5}$ or $(x+3)(x+3)(x-5)$

ex.4) Find a poly of degree 3 w/ zeros $-3, 2, 5$

$f(x)=a(x+3)(x-2)(x-5)$

if $a=1$ then $f(x)=x^3-4x^2-11x+30$

ex.5) Identify zeros & multiplicity of

$f(x)=5x^2(x+2)(x-1/2)^4$

$5x^2=0$ → $\frac{5x^2}{5}=\frac{0}{5}$ → $\sqrt{x^2}=\sqrt{0}$ → $\boxed{x=0 \text{ w/ multiplicity of } 2}$

$x+2=0$ → $\boxed{x=-2 \text{ multiplicity of } 1}$

$x-1/2=0$ → $\boxed{x=1/2 \text{ multiplicity of } 4}$

Graphing a poly

ex.6) Graph $f(x) = (x+1)^2(x-2)$

① Find x-int: $(x+1)=0$ $(x-2)=0$

$x=-1$ $x=2$

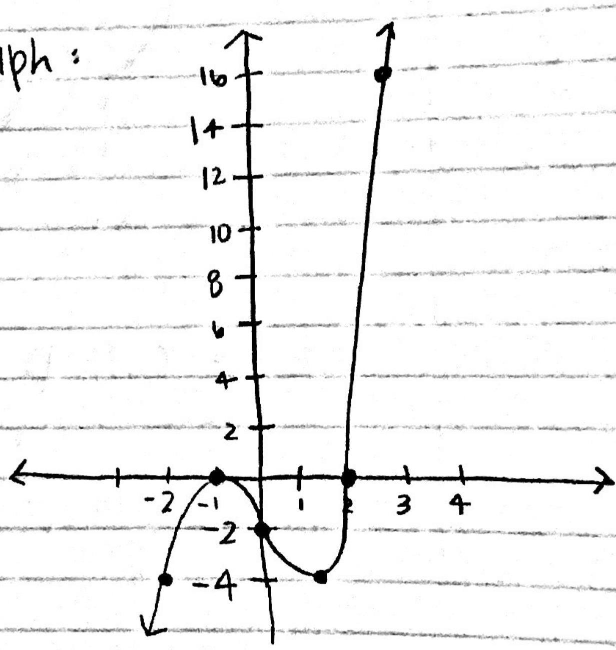
② Find where graph is above or below x-axis:

below x-axis | -1 | below x-axis | 2 | above x-axis

$f(-2)=-4$ $f(1)=-4$ $f(3)=16$

③ Find some other points & graph:

x	
-2	-4
0	-2
1	-4
3	16

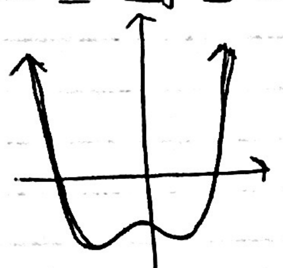
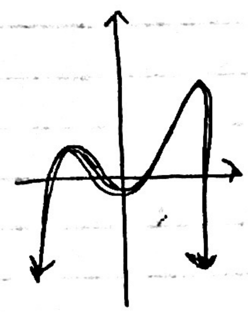
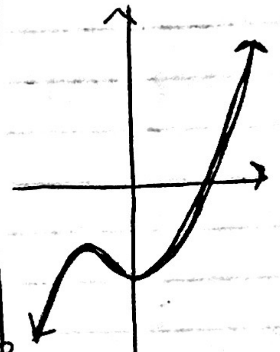
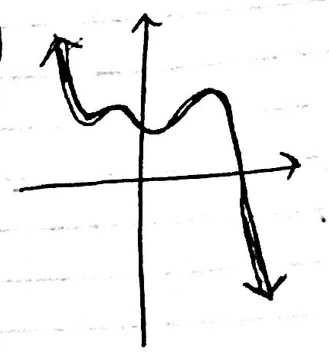


Remember!
* If a zero has even multiplicity then graph touches x-axis.
* If a zero has odd multiplicity it crosses x-axis.

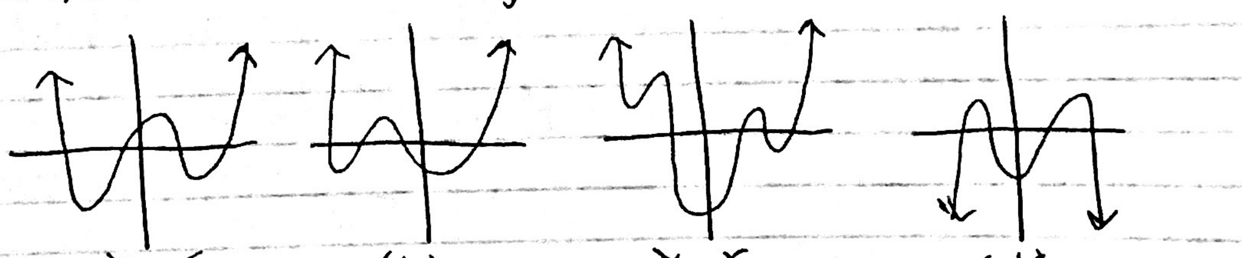
Turning Points

* where a graph changes direction
  → each turning point is a local min or local max.
* If a graph has n-1 turning pts then the degree of f is at least n.

End Behavior

Even Degrees		ODD degrees	
(+) leading coefficient	(−) LC	(+) LC	(−) LC
			
as $x \to -\infty$, $y \to +\infty$ as $x \to +\infty$, $y \to +\infty$	as $x \to -\infty$, $y \to -\infty$ as $x \to +\infty$, $y \to -\infty$	as $x \to -\infty$, $y \to -\infty$ as $x \to +\infty$, $y \to +\infty$	as $x \to -\infty$, $y \to +\infty$ as $x \to +\infty$, $y \to -\infty$

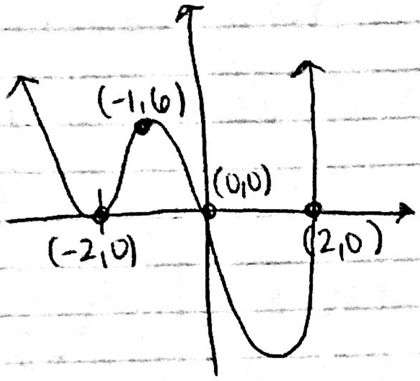
ex B) Which could be the graph of $f(x) = x^4 + ax^3 + bx^2 - 5x - 6$



~~(a)~~ (b) ~~(c)~~ ~~(d)~~

- we know y-int is -6 so (a) is not the graph.
- f is degree 4 so it can have at most 3 turning points so (c) is not the graph.
- since x^4 is a positive leading coefficient it eliminates (d) so $\boxed{\text{(b) is the graph}}$

ex.9) write a poly equation given graph:



x-int: -2, 0, 2
* -2 has even multiplicity
so

$f(x) = ax(x+2)^2(x-2)$

↑ next find a

pick a point: (-1,6)
plug into f(x) → $6 = a(-1)(-1+2)^2(-1-2)$

$6 = a(3)$

$a = 2$

so $\boxed{f(x) = 2x(x+2)^2(x-2)}$

ex.10) Analyze $f(x) = (2x+1)(x-3)^2$

① expand: $f(x) = 2x^3 - 11x^2 + 12x + 9$

② end behavior:

③ x-int: $0 = 2x+1$ $\quad$ $0 = x-3$

$x = -1/2$ $\quad$ $x = 3$

mult. of 1 (crosses) $\quad$ mult. of 2 (touches)

④ y-int: $f(0) = 9$

⑤ max # turning pts: degree 3 has at most 2 turning pts.

⑥ Find additional pts & graph:

$f(-1) = -16$ $\quad$ $(-1,-16)$

$f(1) = 12$ $\quad$ $(1,12)$

$f(4) = 9$ $\quad$ $(4,9)$