Pg.302

4.4 Build Quadratic Models

* optimization – finding max/min values in models

Revenue Formula: $R = xp$ ← price per unit

↑ revenue ↑ # of x units sold

→ an equation that relates p & x is called the demand equation. When the demand eq. is linear, the revenue model is a quadratic function.

ex.1) Maximizing Revenue

*Found that when certain calcs are sold @ price p dollars per unit, the # x of calcs sold is given by the demand eq.:

$$x = 21{,}000 - 150p$$

(a) Find a model that expresses the revenue R as a function of p.

Revenue is $R = xp$ where $x = 21{,}000 - 150p$

so $R = (21{,}000 - 150p)p$

$R = 21{,}000p - 150p^2 = \boxed{150p^2 - 21{,}000p}$

(b) Domain? since x = # of calc sold, $x \geq 0$

so $21{,}000 - 150p \geq 0$

$-21{,}000 \quad\quad -21{,}000$

$\dfrac{-\cancel{150}p}{-\cancel{150}} \geq \dfrac{-21{,}000}{-150}$

$p \leq 140$

also since p = price $p > 0$

$\}$ so, $\boxed{D: \{p \mid 0 < p \leq 140\}}$

(c) What unit price should be used to max revenue?

since $a<0$, vertex is a max so: $a=-150$ $b=21,000$

$$p = \frac{-b}{2a} = \frac{-21,000}{2(-150)} = \boxed{\$70.00}$$

(d) if the price is charged, max revenue is:

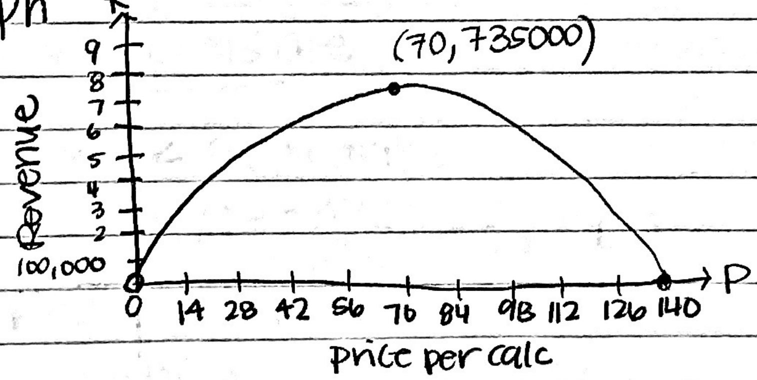
$$R = -150(70)^2 + 21,000(70) = \boxed{\$735,000}$$

(e) How many units sold at this price?

$x = 21,000 - 150p$ at price $p=70$

$x = 21,000 - 150(70) = \boxed{10,500 \text{ calculators}}$

(f) Graph



(g) What price if want \$675,000 in revenue?

$675,000 = -150p^2 + 21,000p$

$$\frac{150p^2 - 21,000p + 675,000}{150} = \frac{0}{150}$$

$p^2 - 140p + 4500 = 0$

$(p-50)(p-90) = 0$

$p=50$ $p=90$

should charge between $\boxed{\$50 \ \& \ \$90}$

ex2) Maximize Area enclosed by fence

- Farmer has 2000 yds of fence to enclose rectangle field. Find dimensions for max area.

Perimeter: let x be length & w width.

$$2x+2w=2000 \leftarrow \text{solve for } w: \quad \frac{\cancel{2}w}{\cancel{2}}=\frac{2000-2x}{2}$$

Area: $A = x \cdot w$

$$w = 1000 - x$$

$$A = x(1000-x) = -x^2+1000x$$

*Now Find Max: $a=-1$ $b=1000$

$$x = \frac{-b}{2a} = \frac{-1000}{2(-1)} = \frac{-1000}{-2} = 500 \leftarrow \text{length}$$

$$A(500) = -(500)^2+1000(500) = \underset{\uparrow \text{area}}{250{,}000}$$

$\boxed{\text{dimensions are 500 by 500 yds}}$

ex.3) Look @ book for scenario:

$$h(x) = \frac{-32x^2}{(400)^2}+x+500$$

(a) max height ↓ simplify

$$h(x) = \frac{-1}{5000}x^2+x+500$$

$$x = \frac{-1}{2\left(-\frac{1}{5000}\right)} = \frac{5000}{2} = 2500 \leftarrow \text{horiz. distance from base}$$

$$h(2500) = \frac{-1}{5000}(2500)^2+2500+500 = \boxed{1750 \text{ ft.} \ \uparrow \text{height}}$$

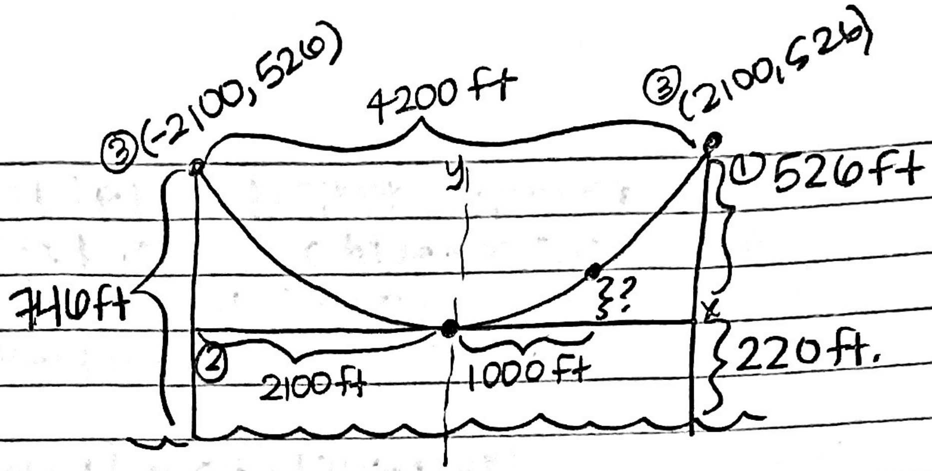
(b) How far from base to strike the water

$h(x) = \text{height} = 0$

$$0 = \frac{-1}{5000}x^2+x+500 \qquad x = \frac{-1 \pm \sqrt{1^2-4\left(-\frac{1}{5000}\right)(500)}}{2\left(-\frac{1}{5000}\right)}$$

$x = -4\cancel{58}$, $\boxed{5458 \text{ ft.}}$

ex.4)



④ Find equation of line given vertex @ (0,0) & a pt @ (2100,526)

h k   x y

$y = a(x-h)^2 + k$

$526 = a(2100-0)^2 + 0$

$526 = a(2100)^2$

$a = \frac{526}{(2100)^2}$

So equation is:

$y = \frac{526}{(2100)^2}(x-0)^2 + 0$

$y = \frac{526}{(2100)^2}(x)^2$

⑤ when x = 1000 ft, the height is:

$y = \frac{526}{(2100)^2}(1000)^2 = \boxed{119.3 \text{ ft.}}$