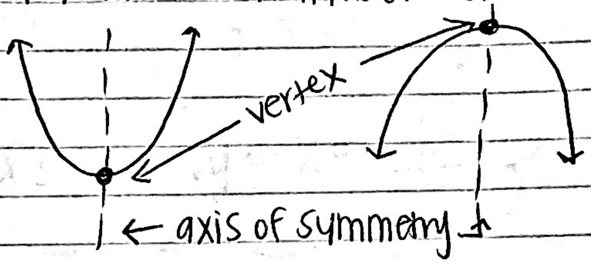
pg. 291 HW: #9, 13–19 odd, 33–61 odd, 81, 83

4.3 Quadratic Functions & their properties

* <u>Quadratic Function</u>: A function in the form

$$f(x) = ax^2 + bx + c$$

where a, b, c are real numbers and $a \neq 0$.



<u>Transformations w/ Quadratics</u>

* You can use the method of completing the square to put the quadratic into the form:

$$f(x) = a(x-h)^2 + k$$

EX. 1) Graph $f(x) = 2x^2 + 8x + 5$

① $f(x) = 2(x^2+4x)+5$ ← Factor out the 2

② $f(x) = 2(x^2+4x+4)+5-8$ ← complete the square

③ $2(x+2)^2 - 3$

vertex: $(-2,-3)$

of symmetry: $x=-2$

So if in the form: $f(x) = a(x-h)^2+k$

→ vertex: (h,k)

→ axis of symmetry: $x=h$

→ graph opens up if $a>0$

→ graph opens down if $a<0$

* if quadratic is in the form:

$$f(x) = ax^2 + bx + c$$

$$\text{Vertex} = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

$$\text{Axis of symmetry} = x = \frac{-b}{2a}$$

ex.2) locate vertex & AOS for $f(x) = -3x^2 + 6x + 1$
Does it open down or up?

vertex: $h = -\frac{b}{2a} = -\frac{6}{2(-3)} = 1$

$k = f\left(-\frac{b}{2a}\right) = f(1) = -3(1)^2 + 6(1) + 1 = 4$

$\boxed{\text{vertex}: (1,4)}$ $\boxed{\text{axis of symm.}: x = 1}$

$\boxed{\text{opens down}}$ since $a = -3$ and $a < 0$

X-int in Quadratics

* $b^2 - 4ac \rightarrow$ discriminant

so if ① $b^2 - 4ac > 0$, the graph has 2 x-int.
② $b^2 - 4ac = 0$, the graph has 1 x-int.
③ $b^2 - 4ac < 0$, the graph has <u>NO</u> x-int.

* use the quadratic formula to find x-int.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex.3) given $f(x) = -3x^2 + 6x + 1$

ⓐ Find intercepts: $a = -3$ $b = 6$ $c = 1$

discriminant is greater than 0, so I will have 2 x-int.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-3)(1)}}{2(-3)} = \frac{-6 \pm \sqrt{48}}{-6}$$

$$x = \frac{-6 + \sqrt{48}}{-6} \qquad x = \frac{-6 - \sqrt{48}}{-6}$$

$\boxed{x = -0.15}$ $\boxed{x = 2.15}$

y-int: let $x = 0$

$y = -3(0)^2 + 6(0) + 1$

$\boxed{y = 1}$

ⓑ graph & Find Domain & Range:

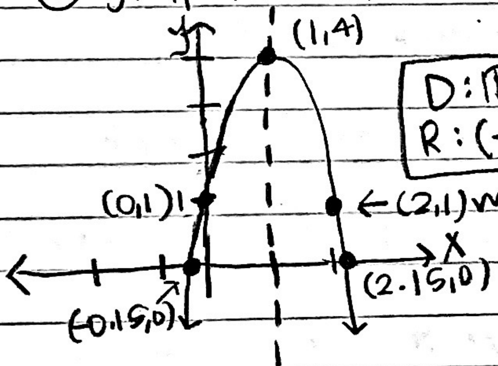
From example 2 we know:

vertex : $(1, 4)$

AOS : $x = 1$

D: $\mathbb{R}$ or $(-\infty, +\infty)$

R: $(-\infty, 4]$ or $\{y \mid y \le 4\}$



↙ use x-values

ⓒ inc | dec: inc: $(-\infty, 1)$

dec: $(1, +\infty)$

ex.4) graph $f(x) = x^2 - 6x + 9$ (we need vertex, AOS, int.)

↑ since $a = 1 > 0$, opens up!

vertex: $h = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$

$k = (3)^2 - 6(3) + 9 = 0$

so vertex: $(3, 0)$ ↖ also x-int!

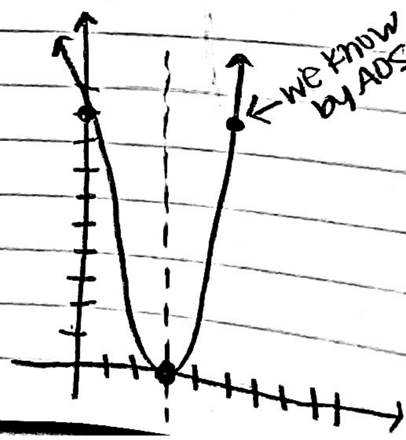
AOS: $x = 3$

y-int: when $x = 0$ so $y = 0^2 - 6(0) + 9 = 9$

$\boxed{(0, 9)}$

D: $\mathbb{R}$ R: $[0, +\infty)$

inc: $(3, +\infty)$ dec: $(-\infty, 3)$



Graph Quadratic given vertex & other pt.

use: $\underbrace{f(x)}_{y}=a(x-h)^2+k$

ex.6) Find quadratic function whose
vertex: $(1,-5)$ and y-int @ -3.

$\underset{h}{1}$, $\underset{k}{-5}$ $\quad (\underset{x}{0},\underset{y}{-3})$

$f(x)=a(x-1)^2+(-5)$

$f(x)=a(x-1)^2-5$ ← now sub in to find a

$-3=a(0-1)^2-5$

$-3=a-5$

$a=2$ ← plug back into $f(x)=a(x-1)^2-5$

$f(x)=2(x-1)^2-5=\boxed{2x^2-4x-3}$

Find Max/Min value

Max/Min value wants the y-value of max or min

so if $a>0$ → graph opens up → min value @ $f\left(-\frac{b}{2a}\right)$

$a<0$ → graph opens down → max value @ $f\left(-\frac{b}{2a}\right)$

ex.7) Find max or min value of $f(x)=x^2-4x-5$

$a=1 \quad b=-4 \quad c=-5$

$a=1>0$ so it will have a $\boxed{\text{min}}$

① Find $x=\frac{-b}{2a}$ first: $x=\frac{-(-4)}{2(1)}=2$

② Plug into f(x): $f(2)=2^2-4(2)-5=-9$ ← $\boxed{\text{min value @ } -9}$