

3.5 Graphing Techniques : Transformations

HW: 5-35 odd, 39-65 odd

Vertical/Horizontal Shifts

Vertical Shift:

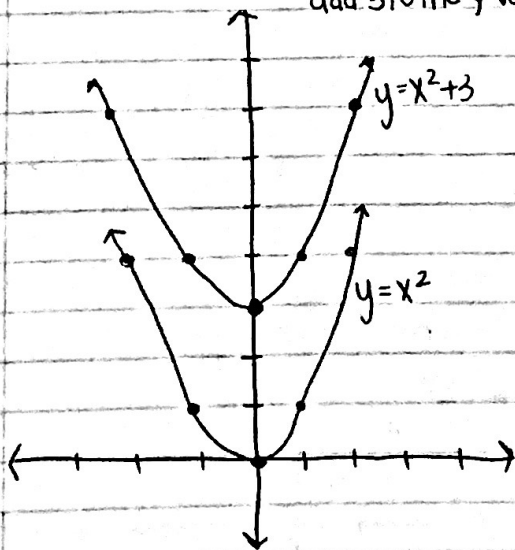
UP

$y = f(x) + k$ is the graph of f shifted vertically up k units

ex. 1) given $f(x) = x^2$

x	$y = x^2$	$y = x^2 + 3$
-2	4	7
-1	1	4
0	0	3
1	1	4
2	4	7

add 3 to the y-values



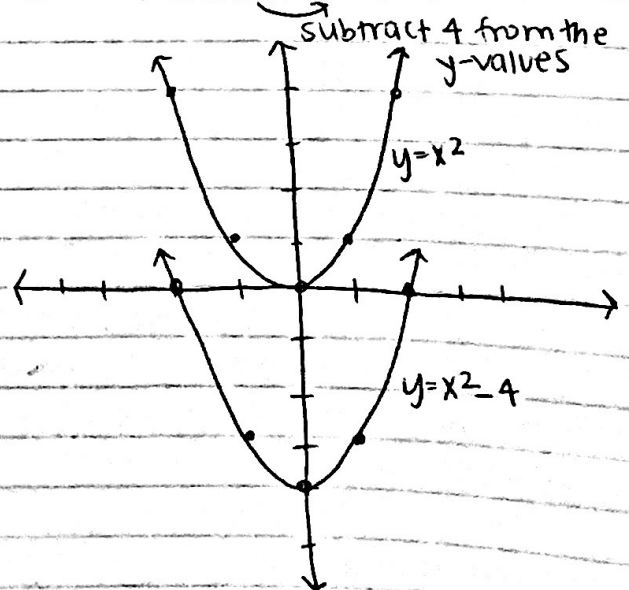
DOWN

$y = f(x) - k$ is the graph of f shifted down k units

ex. 2) given $f(x) = x^2$

x	$y = x^2$	$y = x^2 - 4$
-2	4	0
0	0	-4
1	1	-3
2	4	0
-1	1	-3

subtract 4 from the y-values



Horizontal Shift:

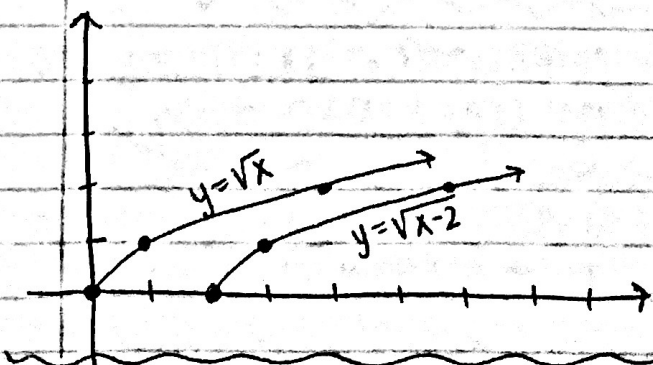
Right

$y = f(x-h)$ is the graph of f shifted horizontally right h units.

ex. 3) given $f(x) = \sqrt{x}$

$y = \sqrt{x}$		$y = \sqrt{x-2}$	
x	y	x	y
0	0	2	0
1	1	3	1
4	2	6	2
9	3	11	3
16	4	18	4

↗
add 2 to each x-value



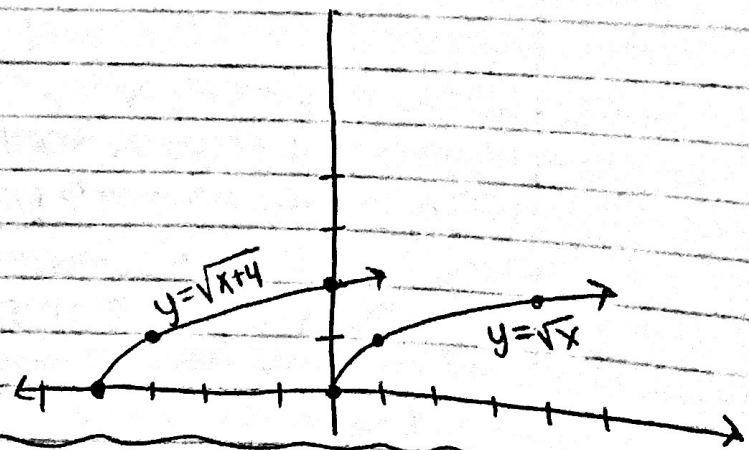
Left

$y = f(x+h)$ is the graph of f shifted horizontally left h units.

ex. 4) given $f(x) = \sqrt{x}$

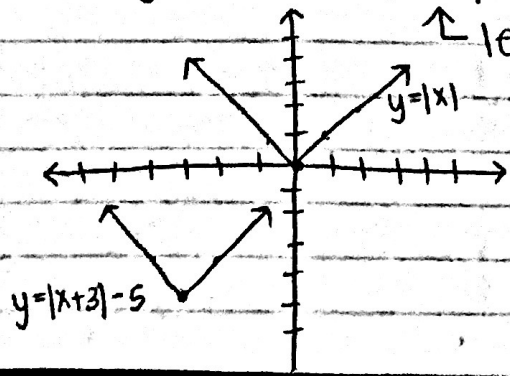
$y = \sqrt{x}$		$y = \sqrt{x+4}$	
x	y	x	y
0	0	-4	0
1	1	-3	1
4	2	0	2
9	3	6	3

↖
subtract 4 from each x-value



Combining Vertical & Horizontal

ex. 5) graph $f(x) = |x+3| - 5$ ← down 5 (subtract 5 from y)



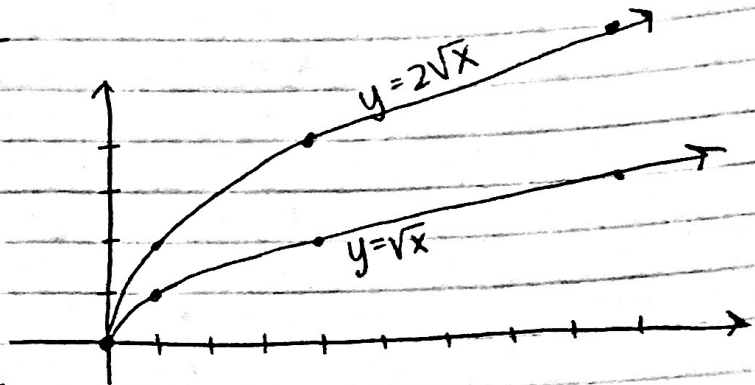
↖
left 3 (subtract 3 from x)

$ x $		$ x+3 - 5$	
x	y	x	y
-2	2	-5	-3
-1	1	-4	-4
0	0	-3	-5
1	1	-2	-4
2	2	-1	-3

Compressions & Stretches

ex. 6) vertical stretch

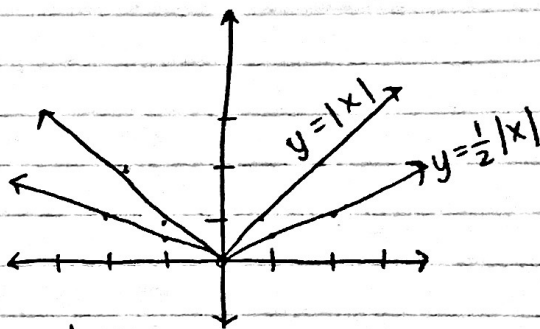
x	$y = \sqrt{x}$	$y = 2\sqrt{x}$
0	0	0
1	1	2
4	2	4
9	3	6



multiply y-value by 2

ex. 7) vertical compression

x	$y = x $	$y = \frac{1}{2} x $
-2	2	1
-1	1	1/2
0	0	0
1	1	1/2
2	2	1



multiply y-value by 1/2

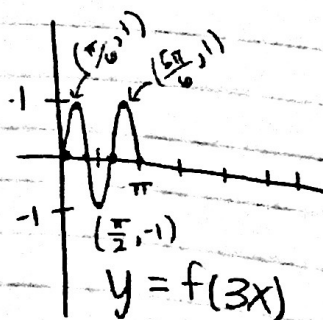
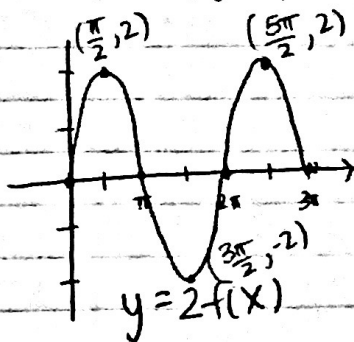
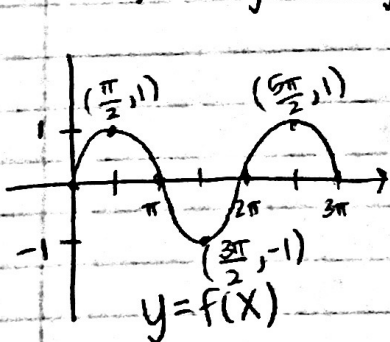
multiply y-values by "a"

$y = af(x)$ is the graph of f that is vertically compressed when $0 < a < 1$ and vertically stretched when $a > 1$.

multiply x-values by "(1/a)"

$y = f(ax)$ is the graph of f that is a horizontal compression when $a > 1$ and stretch when $0 < a < 1$.

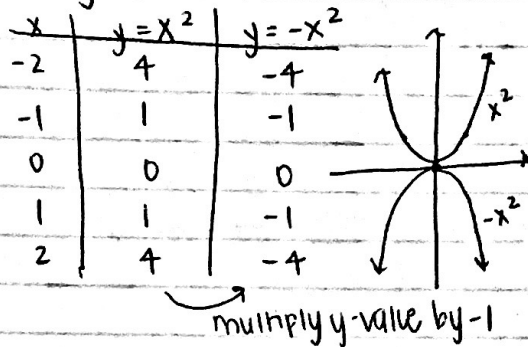
ex. 9) the graph $y = f(x)$ is given, graph $f(3x)$ and $2f(x)$.



Reflection

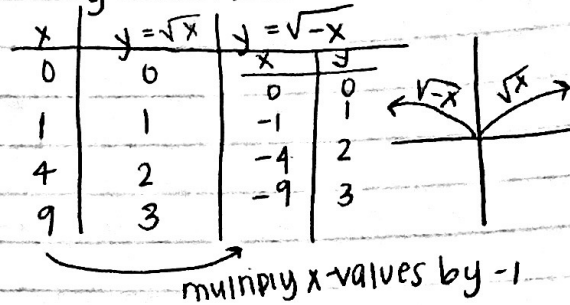
over x-axis

$$y = -f(x)$$



y-axis

$$y = f(-x)$$



Summary

Vertical shifts $\begin{cases} y = f(x) + k & \text{move } f \text{ up by } k \text{ units} \\ y = f(x) - k & \text{move } f \text{ down by } k \text{ units} \end{cases}$

Horizontal shifts $\begin{cases} y = f(x+h) & \text{move } f \text{ left by } h \text{ units} \\ y = f(x-h) & \text{move } f \text{ right by } h \text{ units} \end{cases}$

compress or stretch $\begin{cases} y = af(x) & \text{multiply each } y\text{-value by "a"} \\ y = f(ax) & \text{multiply each } x\text{-value by "1/a"} \end{cases}$

$\begin{cases} \text{vertical stretch if } a > 1 \\ \text{vertical compress if } 0 < a < 1 \\ \text{horiz. stretch if } 0 < a < 1 \\ \text{horiz. compress if } a > 1 \end{cases}$

reflect x-axis $\begin{cases} y = -f(x) & \text{reflect } f \text{ over } x\text{-axis (mult. } y \text{ by } -1) \end{cases}$

reflect y-axis $\begin{cases} y = f(-x) & \text{reflect } f \text{ over } y\text{-axis (replace } x \text{ by } -x) \end{cases}$

PRACTICE: ex. 11) write an equation that has the following transformation given the parent $f(x) = |x|$

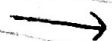
1. left 2 2. up 3 3. reflect over y-axis

$$y = |-x + 2| + 3$$

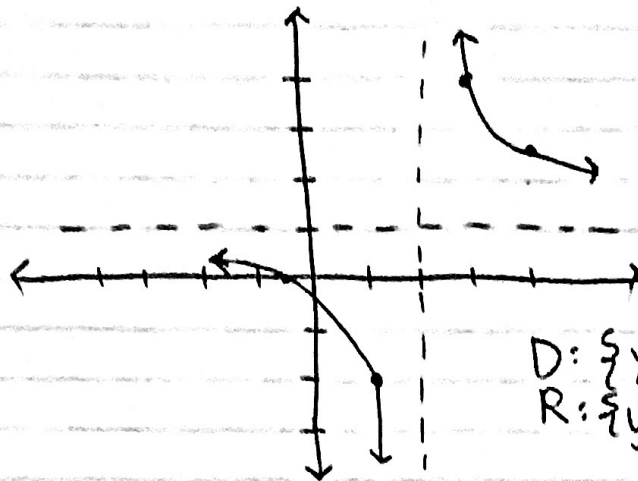
ex. 12) Graph $f(x) = \frac{3}{x-2} + 1$

① Find points for $\frac{1}{x} : (1, 1) (2, \frac{1}{2}) (-1, -1)$

② multiply y-value by 3: $(1, 3) (2, \frac{3}{2}) (-1, -3)$



- ③ Add 2 to each x-value: $(3,3)(4, \frac{3}{2})(1,-3)$
 ④ Add 1 to each y-value: $(3,4)(4, \frac{5}{2})(1,-2)$



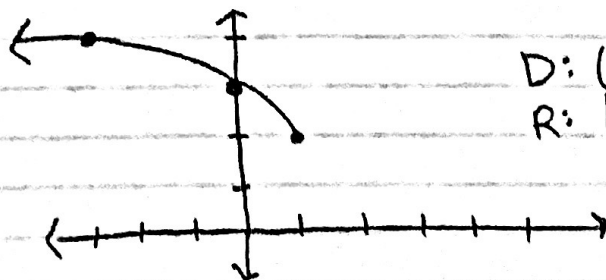
$D: \{x | x \neq 2\}$
 $R: \{y | y \neq 1\}$

← since we moved right 2
 ↑ since we moved up 1

ex.13) Graph $f(x) = \sqrt{1-x} + 2$

- ① Rewrite: $y = \sqrt{-(x-1)} + 2$
 ② Find points for $y = \sqrt{x}$: $(0,0)(1,1)(4,2)$
 ③ change x to -x: $(0,0)(-1,1)(-4,2)$
 ④ add 1 to x-value: $(1,0)(0,1)(-3,2)$
 ⑤ add 2 to y-value: $(1,2)(0,3)(-3,4)$

*** IMPORTANT!**
 Since horizontal shift must be in form $(x-h)$ you must rewrite problem if it is not



$D: (-\infty, 1]$
 $R: [2, +\infty)$