3.3 Properties of Functions

Even & ODD functions

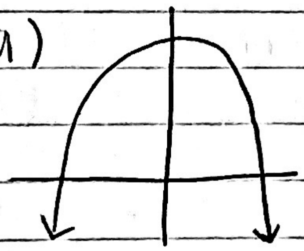
* Even: a function is even iff whenever the point (x, y) is on the graph, the point $(-x, y)$ is also on the graph
* ODD: a function is odd iff whenever the point (x, y) is on the graph, $(-x, -y)$ is also on the graph.

Theorem: A function is even iff the graph is symmetric w/ respect to the y-axis. A function is odd iff its graph is symmetric w/ respect to the origin.

Summary

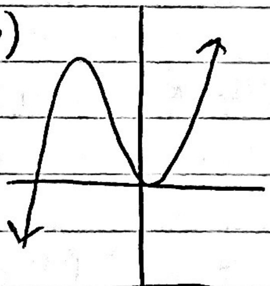
Even	ODD
* $f(-x) = f(x)$	* $f(-x) = -f(x)$
* symmetric over y-axis	* symmetric over origin

ex.1) (a)



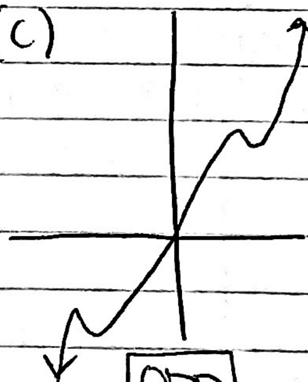
$\boxed{\text{even}}$

(b)



$\boxed{\text{neither}}$

(c)



$\boxed{\text{ODD}}$

ex.2) (a) $f(x) = x^2 - 5$

$f(-x) = (-x)^2 - 5 = x^2 - 5 = f(x)$

$f(-x) = f(x)$ so $\boxed{\text{even}}$

(b) $g(x) = x^3 - 1$

$g(-x) = (-x)^3 - 1 = -x^3 - 1 \neq g(x) \neq -g(x)$ $\boxed{\text{neither}}$

(c) $h(x) = 5x^3 - x$

$h(-x) = 5(-x)^3 - (-x) = -5x^3 + x = -(5x^3 - x) = -h(x)$

$\boxed{\text{ODD}}$

(d) $F(x) = |x|$

$F(-x) = |-x| = |x| = F(x)$ $\boxed{\text{even}}$

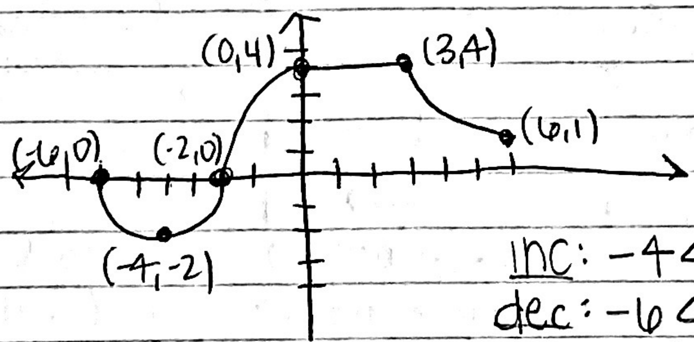
→ Try #37-47 odd

Increasing / Decreasing / Constant

<u>increasing</u>: as x-values get bigger y-values get bigger

<u>decreasing</u>: as x-values get ~~sma~~ bigger y-values get smaller

<u>constant</u>: as x-values get bigger y-values stay the same



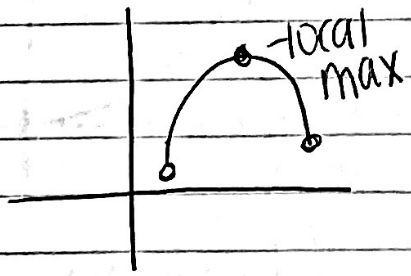
<u>inc</u>: $-4 < x < 0$ or $(-4, 0)$

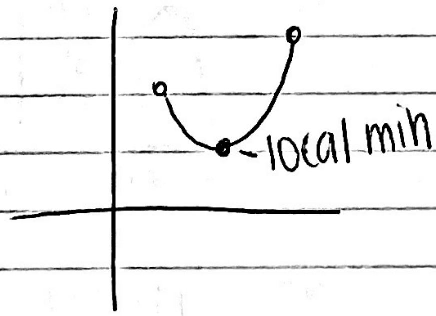
<u>dec</u>: $-6 < x < 4$ and $3 < x < 6$

$(-6, 4)$ & $(3, 6)$

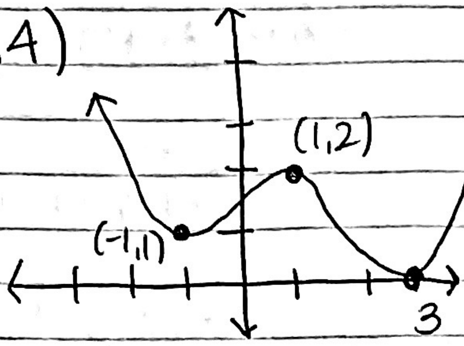
<u>constant</u>: $0 < x < 3$ or $(0, 3)$

Local Max / Min





ex.4)

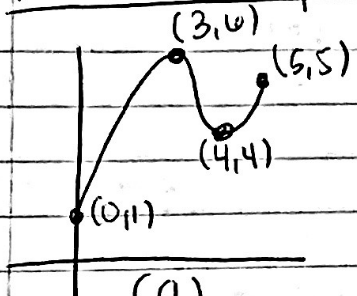


(a) local max: (1,2)

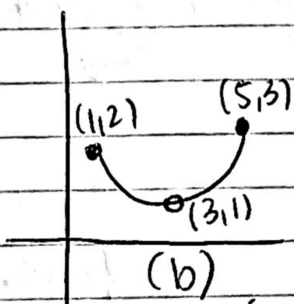
(b) local min: (-1,1), (3,0)

(c) inc: $-1<x<1$ & $x>3$ or
$(-1,1)$ & $(3,+\infty)$
dec: $x<-1$ & $1<x<3$ or
$(-\infty,-1)$ & $(1,3)$

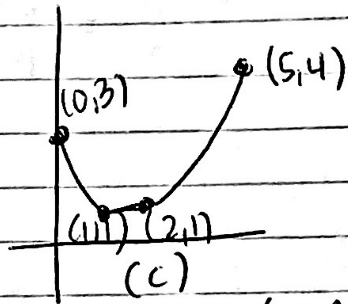
Absolute Max/Min



(a)



(b)

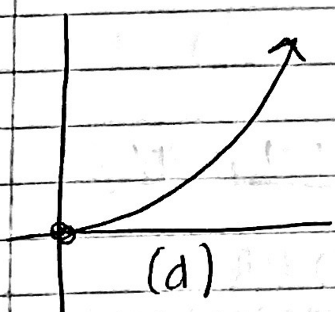


(c)

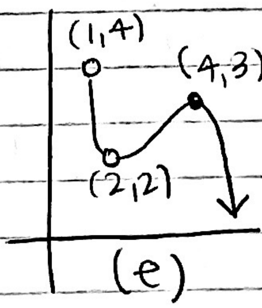
ab.max: (3,6)
ab.min: (0,1)
loc.max: (3,6)
loc.min: (4,4)

ab.max: (5,3)
ab.min: none
loc.max: none
loc.min: none

ab.max: (5,4)
ab.min: occurs @ y=1
loc.max: none
loc.min: occurs @ y=1
any # in the interval [1,2]



(d)



(e)

ab.max: none
ab.min: (0,0)
local max: none
loc.min: none

ab.max: none
ab.min: none
loc.max: (4,3)
loc.min: none

* important:
ab.max/min can be an endpoint, but local max/min cannot be an endpoint.

Average Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}, \quad a \neq b$$

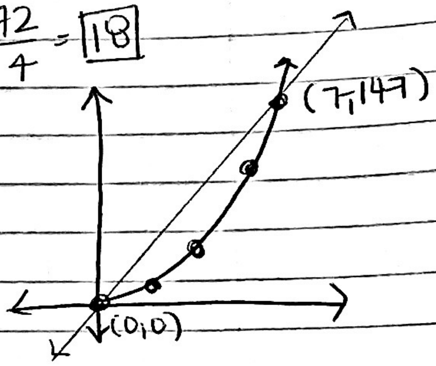
ex. 7) Find ARoC of $f(x)=3x^2$

(a) from 1-3

$a=1$, $b=3$ $\quad \frac{f(3)-f(1)}{3-1} = \frac{[3(3)^2]-[3(1)^2]}{3-1} = \frac{27-3}{3-1} = \frac{24}{2} = \boxed{12}$

(b) from 1 to 5

$a=1$, $b=5$ $\quad \frac{f(5)-f(1)}{5-1} = \frac{75-3}{5-1} = \frac{72}{4} = \boxed{18}$



Secant Line

$$m_{sec} = \frac{f(b)-f(a)}{b-a}$$

*Theorem: the average rate of change of a function from a to b equals the slope of the secant line containing the two points $(a, f(a))$ and $(b, f(b))$ on its graph

Finding equation of Secant Line

Given $g(x)=3x^2-2x+3$

(a) Find av. rate of change from -2 to 1

$$\frac{g(1)-g(-2)}{1-(-2)} = \frac{4-19}{3} = \frac{-15}{3} = \boxed{-5}$$

(b) Find equation of secant line containing $(-2, g(-2))$ & $(1, g(1))$

$(-2, 19)$ $\quad (1, 4)$

*use point slope form: $y-y_1 = m_{sec}(x-x_1)$

$y-19 = -5(x-(-2))$

$y-19 = -5x-10$

$\boxed{y=-5x+9}$