Teacher Notes

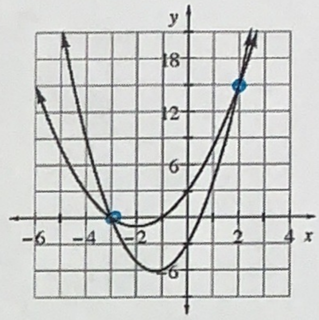
3.2.1 How can I solve inequalities?

Solving Inequalities with One or Two Variables

In this chapter, you have developed many strategies for solving equations with one variable and solving systems of equations with two variables. But what if you want to solve an inequality or system of inequalities instead? Today you will explore how to use familiar strategies to solve inequalities.

3-67.

In the previous section, you learned how to use the graph of a system to solve an equation. How can the graphs of $y = 2x^2 + 5x - 3$ and $y = x^2 + 4x + 3$ (shown at right) help you solve an *inequality*? Consider this as you complete the parts below.



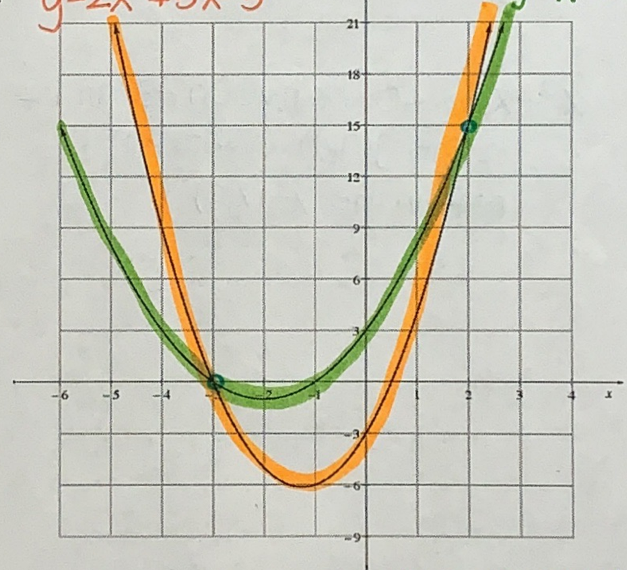
a. How can you use the graph to determine the solutions to $2x^2 + 5x - 3 = x^2 + 4x + 3$? What are the solutions?

Solutions are where the graphs intersect.

$\boxed{x=-3}$ $\boxed{x=2}$

b. label each graph with its equation and highlight each function with a different color. How did you decide which graph matches which function?

$y=2x^2+5x-3$ $\quad$ $y=x^2+4x+3$

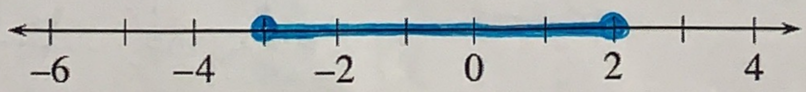


less than or equal to

c. Use the graph to identify the x-values for which $2x^2 + 5x - 3 \leq x^2 + 4x + 3$. How did you locate the solutions? How many solutions are there? How can you describe all of the solutions?

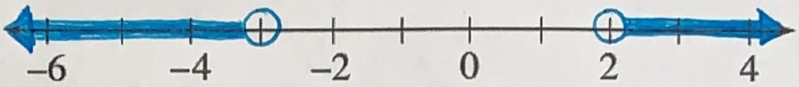
The solutions are where $2x^2+5x-3$ is <u>BELOW</u> x^2+4x+3.
Everything between $x=-3$ to $x=2$ or $-3 \leq x \leq 2$.

d. How can these solutions be represented on a number line? Locate the number line labeled $2x^2 + 5x - 3 \leq x^2 + 4x + 3$ on your resource page. Use a colored marker to highlight the solutions to the inequality on the number line.



greater than everywhere it wasn't ≤

e. What about the inequality $2x^2 + 5x - 3 > x^2 + 4x + 3$? What are the solutions to this inequality? Represent your solutions algebraically and on a number line.



3-69.

Bert and Ernie are solving the inequality $2x^2 + 5x - 3 < x^2 + 4x + 3$. They are looking at the graph in problem 3-67 when Bert has an idea. "*Can't we change this into one parabola and solve our inequality that way?*", he asks.

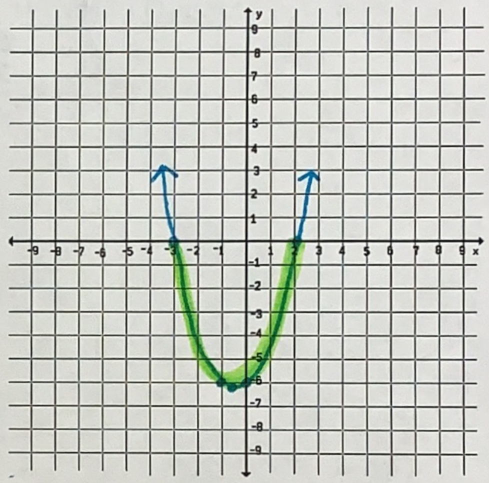
Ernie asks, "*What do you mean?*"

"*Can't we determine the solutions by looking at the graph of* $f(x) = x^2 + x - 6$*?*", Bert replies.

a. Where does Bert get the equation $f(x) = x^2 + x - 6$?

moves all terms to one side.

b. Try Bert's idea. Make a sketch of the parabola and show how it can be used to solve the original inequality.

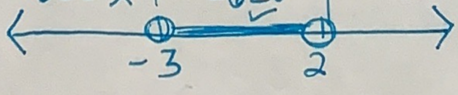


$x^2+x-6<0$ between −3 and 2 on the graph (where it is below the x-axis)

c. "*Just a minute!*" mumbles Ernie, "*I think I have another method. Instead of graphing the parabola, can't we just rewrite the new inequality as* $x^2 + x - 6 < 0$ *and then solve the equation* $x^2 + x - 6 = 0$*? This would give us the boundary points and then we could test numbers in the original inequality to see the regions that contain the solutions.*" Use Ernie's method to solve the inequality. Does it give the same solution?

$(-4)^2-4-6<0$ | $0^2+0-6<0$ | $3^2+3-6<0$

$6<0$ X | $-6<0$ ✓ | $6<0$ X



$x^2+x-6=0$

(Factor)

$(x+3)(x-2)=0$

$x=-3 \quad x=2$

boundary points

d. Use any method to solve the inequality $x^2 - 3x - 10 \geq 0$. sketch a graph

greater than means it is above the x-axis

$\boxed{\text{when } x \leq -2 \text{ or } x \geq 5}$