3.1 Functions

* *Relation* - correspondance between 2 sets
* *Domain* - set of all inputs for a relation
* *Range* - set of all outputs

* *Function* - a relationship that associates w/ each element of X exactly one element of Y

Figure 3 in book → No single answer to the Q "What is colleen's phone #?" = not a function

Figure 4 ⟶ could answer "What is life expectancy of dog?" "What is life expectancy of Kang?" = function

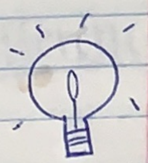
Figure 7 → Domain: {Citgo, Shell, Texaco, Valero} Range: {\$3.19, \$3.29, \$3.35} = function

Figure 8 → Domain: {0.70, 0.71, 0.75, 0.78} Range: {\$1529, \$1575, \$1765, \$1798, \$1952} = not function

#19 on pg.211

Elvis → Jan 8
Colleen → Jan 8
Kaleigh → Mar 15
Marissa → Sept. 17

D: {Elvis, Colleen, Kaleigh, Marissa}
R: {Jan 8, Mar. 15, Sept. 17}
= Function



Idea behind a function is it's predictability
"nonfunctions" we don't have the predictability

ex.3) (a) {(1,4), (2,5), (3,6), (4,7)}

$\boxed{\begin{array}{l} D: \{1,2,3,4\} \\ R: \{4,5,6,7\} \end{array}}$ $\boxed{\text{Function}}$

x = independent variable
y = dependent variable

(b) $\{(1,4),(2,4),(3,5),(6,10)\}$

$\boxed{\begin{array}{ll} D: \{1,2,3,6\} & \text{Function} \\ R: \{4,5,10\} & \end{array}}$

(c) $\{(-3,9),(-2,4),(0,0),(1,1),(-3,8)\}$

$\boxed{\begin{array}{ll} D: \{-3,-2,0,1\} & \text{Not a function} \\ R: \{0,1,4,8,9\} & \end{array}}$

Finding values of a function

ex. 6) $f(x) = 2x^2 - 3x$

(a) $f(3) = 2(3)^2 - 3(3) = 9$

(b) $f(x) + f(3) = 2x^2 - 3x + 9$

(c) $3f(x) = 3(2x^2 - 3x) = 6x^2 - 9x$

(d) $f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$

(e) $-f(x) = -(2x^2 - 3x) = -2x^2 + 3x$

(f) $= f(3x) = 2(3x)^2 - 3(3x) =$

$2(9x^2) - 9x = 18x^2 - 9x$

(g) $f(x+3) = 2(x+3)^2 - 3(x+3)$

$2(x^2 + 6x + 9) - 3x - 9$

$2x^2 + 12x + 18 - 3x - 9$

$2x^2 + 9x + 9$

→ Try #19-30 odd, #43-49 odd

Difference Quotient: $\dfrac{f(x+h) - f(x)}{h}$

*used in calc to define derivative, which applies to velocity & optimization

ex. 8) (a) $f(x) = 2x^2 - 3x$ $\quad \dfrac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$

$\dfrac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h} = \cancel{2x^2} + 4xh + 2h^2 \cancel{-3x} - 3h \cancel{-2x^2} \cancel{+3x}$

$\dfrac{4xh + 2h^2 - 3h}{h} = \dfrac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} = \boxed{4x + 2h - 3}$

(b) $f(x) = \frac{4}{x}$ $\quad \dfrac{\frac{4}{x+h} - \frac{4}{x}}{h} = \dfrac{\frac{4x}{x^2+hx} - \frac{(4x+4h)}{x^2+hx}}{h}$

$\dfrac{\frac{-4h}{x^2+hx}}{h}$ or $\frac{-4h}{x^2+hx} \div \frac{h}{1} = \frac{-4h}{x^2+hx} \cdot \frac{1}{h} = \frac{-4\cancel{h}}{\cancel{h}(x^2+hx)} = \boxed{\frac{-4}{x^2+hx}}$

(c) $f(x) = \sqrt{x}$ $\quad \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$ ← rationalize numerator to get rid of $\sqrt{\ }$ in num.

Diff of squares
$a^2 - b^2 = (a-b)(a+b)$

$\dfrac{(\sqrt{x+h} - \sqrt{x})}{h} \dfrac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \dfrac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \dfrac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$

multiply by conjugate

$\dfrac{h}{h(\sqrt{x+h} + \sqrt{x})} = \boxed{\dfrac{1}{\sqrt{x+h} + \sqrt{x}}}$

→ Try supplemental Problems

Find Domain of Function

(a) $f(x) = x^2 + 5x$ $\boxed{\text{all real \#s}}$

(b) $g(x) = \dfrac{3x}{x^2-4}$ ← Denom cannot $= 0$ so $\boxed{D: \{x \mid x \neq -2, 2\}}$

(c) $= h(t) = \sqrt{4-3t}$ ← can't have (−) neg under $\sqrt{\ }$ so

$4 - 3t \geq 0$
$-4 \quad\quad -4$
$-3t \geq -4$
$t \leq \frac{4}{3}$

$\boxed{D: \{t \mid t \leq \tfrac{4}{3}\} \text{ or } (-\infty, \tfrac{4}{3}]}$

(d) $F(x) = \dfrac{\sqrt{3x+12}}{x-5}$ ← $3x + 12 \geq 0$, $x \geq -4$
← $x \neq 5$

$\{x \mid x \geq -4, x \neq 5\}$

→ Try # 51 − 65 odd

Operations on Functions

* $(f+g)(x) = f(x) + g(x)$
* $(f-g)(x) = f(x) - g(x)$
* $(f \cdot g)(x) = f(x) \cdot g(x)$

Domain is the #s x in both f(x) & g(x)

* $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

D: $\{x \mid g(x) \neq 0\}$

ex. 11) $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{x}{x-1}$

(a) $(f+g)(x) = \frac{1}{x+2} + \frac{x}{x-1} = \frac{(x-1)(1)}{(x-1)(x+2)} + \frac{(x)(x+2)}{(x-1)(x+2)}$

$\frac{x-1+x^2+2x}{(x-1)(x+2)} = \boxed{\frac{x^2+3x-1}{(x-1)(x+2)}}$ D: $\{x \mid x \neq -2, 1\}$

(b) $(f-g)(x) = \frac{x-1}{(x-1)(x+2)} - \frac{(x^2+2x)}{(x-1)(x+2)} = \frac{x-1-x^2-2x}{(x-1)(x+2)}$

$\boxed{\frac{-x^2-1x-1}{(x-1)(x+2)}}$ D: $\{x \mid x \neq 1, -2\}$

same

(c) $(f \cdot g)(x) = \frac{1}{x+2} \cdot \frac{x}{x-1} = \boxed{\frac{x}{(x+2)(x-1)}}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{1}{x+2} \cdot \frac{x-1}{x} = \boxed{\frac{x-1}{x(x+2)}}$ D: $\{x \mid x \neq -2, 0, 1\}$

HW: 67. $f(x) = 3x+4$ $g(x) = 2x-3$

a) $(f+g)(x) = 5x+1$

97. $A = (x)\left(\frac{x}{2}\right)$ so $\boxed{A = \frac{x^2}{2}}$