13.3 Geometric sequences

GEOMETRIC SEQUENCE is a sequence when the ratio of successive terms is always the same nonzero numer

r = common ratio

$r=\frac{54}{18}=3$ $r=\frac{162}{54}=3$

ex.1) Geometric? 2, 6, 18, 54, 162, ...

$r=\frac{6}{2}=3$ $r=\frac{18}{6}=3$

$\boxed{\text{yes! since } r=3}$

ex.2) Geometric? $\{S_n\}=\{2^{-n}\}$. Find 1st term & r.

$S_1=2^{-1}=\frac{1}{2}$ $S_n=2^{-n}$ $S_{n-1}=2^{-(n-1)}$

$$r=\frac{S_n}{S_{n-1}}=\frac{2^{-n}}{2^{-(n-1)}}=2^{-n}(2^{+(n-1)})=2^{-n+(n-1)}=2^{-1}=\frac{1}{2}$$

$\boxed{\text{since } r=\frac{1}{2},\ S_n \text{ is a geometric sequence}}$

ex.3) Geometric? $\{t_n\}=\{3\cdot4^n\}$. Find 1st term & r.

$t_1=3\cdot4^1=12$ $t_n=3\cdot4^n$ $t_{n-1}=3\cdot4^{n-1}$

$$r=\frac{t_n}{t_{n-1}}=\frac{3\cdot4^n}{3\cdot4^{n-1}}=\frac{4^n}{4^{n-1}}=4^n(4^{-(n-1)})=4^{n-(n-1)}=4$$

$\boxed{\text{since } r=4,\ t_n \text{ is a geometric sequence}}$

FIND THE NTH TERM OF A GEOMETRIC SEQUENCE

$$\boxed{a_n = a_1 r^{n-1} \quad r \neq 0}$$

ex.4) $10, 9, \frac{81}{10}, \frac{729}{100}, \ldots$

(a) Find the nth term:

$a_1 = 10$

$r = \frac{9}{10}$

$$\boxed{a_n = 10\left(\frac{9}{10}\right)^{n-1}}$$

(b) Find the 9th term:

$$a_9 = 10\left(\frac{9}{10}\right)^{9-1} = 10\left(\frac{9}{10}\right)^{8} = \boxed{4.3046721}$$

(c) Find a recursive formula:

$$\boxed{a_1 = 10 \quad a_n = \frac{9}{10} a_{n-1}}$$

FIND THE SUM OF THE 1ST N TERMS OF A GEO SEQUENCE

$$\boxed{S_n = a_1 \cdot \frac{1-r^n}{1-r} \quad r \neq 0, 1}$$

ex.5) Find the sum S_n of the 1st n terms of $\left\{\left(\frac{1}{2}\right)^n\right\}$

that is:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \left(\frac{1}{2}\right)^n = \sum_{k=1}^{n} \frac{1}{2}\left(\frac{1}{2}\right)^{k-1}$$

$a_1 = \frac{1}{2}$

$r = \frac{1}{2}$

$$S_n = \frac{1}{2}\left[\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right] = \cancel{\frac{1}{2}}\left[\frac{1-\left(\frac{1}{2}\right)^n}{\cancel{\frac{1}{2}}}\right] = \boxed{1-\left(\frac{1}{2}\right)^n}$$

Determine whether a Geosequence Converges or Diverges

If $|r|<1$, the infinite geometric series

$\sum_{k=1}^{\infty} a_1 r^{k-1}$ converges. and its sum is

$$\boxed{\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1-r}}$$

if it doesnt converge then it diverges.

ex.7) converge or diverge? If converge, find sum.

$$\sum_{k=1}^{\infty} 2\left(\frac{2}{3}\right)^{k-1} = 2 + \frac{4}{3} + \frac{8}{9} + \ldots$$

$a_1 = 2$, $r = \frac{2}{3}$ since $|r|<1$ it $\boxed{\text{converges}}$

$$\sum_{k=1}^{\infty} 2\left(\frac{2}{3}\right)^{k-1} = \frac{2}{1-\frac{2}{3}} = \boxed{6}$$

ex.8) Show that $0.\overline{999}\ldots$ equals 1.

$$0.\overline{99} = 0.9 + 0.09 + 0.009 + \ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{100} + \ldots$$

$$= \sum_{k=1}^{\infty} \frac{9}{10^k} = \sum_{k=1}^{\infty} \frac{9}{10 \cdot 10^{k-1}} = \sum_{k=1}^{\infty} \frac{9}{10}\left(\frac{1}{10}\right)^{k-1}$$

$a_1 = \frac{9}{10}$, $r = \frac{1}{10}$

since $|r| = \frac{1}{10} < 1$ it converges so its sum:

$$0.999\ldots = \frac{\frac{9}{10}}{1-\frac{1}{10}} = \frac{\frac{9}{10}}{\frac{9}{10}} = \boxed{1}$$