13.2 Arithmetic sequences

ARITHMETIC SEQUENCE is a sequence of #s in which the difference between each term is constant.

The difference between one term & the next is called the common difference.

Examples:

2, 6, 10, 14, ... The common diff. $d = 4$

15, 13, 11, 9, 7, ... $d = -2$

5, 8, 11, 14, ... $d = 3$

12, 13.5, 15, ... $d = 1.5$

19, 16, 13, ... $d = -3$

ex. 1) Is the sequence 4, 6, 8, 10, ... arithmetic?

yes, since the common difference is 2

ex. 2) Show the sequence is arithmetic

$\{S_n\} = \{3n+5\}$

1st term: $S_1 = 3(1)+5 = 3+5 = 8$

nth term: $S_n = 3n+5$

(n-1)st term: $S_{n-1} = 3(n-1)+5 = 3n+2$

difference d is: $d = S_n - S_{n-1} = (3n+5)-(3n+2) = 5-2 = 3$

*since the common difference of any 2 successive terms is the constant 3, $\{S_n\}$ is arithmetic.

ex.3) show the sequence is arithmetic:

$\{t_n\} = \{4-n\}$

1st term: $t_1 = 4-1 = 3$

nth term: $t_n = 4-n$

$(n-1)$st term: $t_{n-1} = 4-(n-1) = 5-n$

difference: $d = t_n - t_{n-1} = (4-n)-(5-n) = 4-5 = -1$

* $\{t_n\}$ is an arithmetic sequence whose common diff is -1

FINDING the nth TERM

$a_n = a_1 + (n-1)d$ ← difference between terms

a_n: nth term; a_1: 1st term; n: term #

ex.4) Find 41st term of: 2, 6, 10, 14, 18, ...

$a_1 = 2 \quad d = 4 \rightarrow a_{41} = 2+(41-1)4 = \boxed{162}$

ex.5) The 8th term is 75 & 20th term is 39.

(a) 1st term & common difference?

we know $a_8 = 75$ and $a_{20} = 39$

so $\begin{cases} a_8 = a_1 + 7d = 75 \\ a_{20} = a_1 + 19d = 39 \end{cases}$ ② subtract to get rid of a_1

$-12d = 36$

$\frac{-12d}{-12} = \frac{36}{-12}$

$d = -3 \rightarrow$ so $a_1 + 7(-3) = 75$

$a_1 = 96$

$\boxed{a_1 = 96 \text{ and } d = -3}$

(b) Give recursive formula:

$a_1 = 96 \quad a_n = a_{n-1} - 3$

(c) What's the n^{th} term?

$a_n = a_1 + (n-1)d = 96 + (n-1)(-3) = \boxed{99 - 3n}$

FINDING THE SUM

$$\boxed{S_n = \frac{n(a_1 + a_n)}{2}}$$

ex.6) Find the sum of $\{a_n\} = \{3n+5\}$

$$S_n = \frac{n(8 + (3n+5))}{2} = \boxed{\frac{n(3n+13)}{2}}$$

Ex.7) Find the sum: $60 + 64 + 68 + 72 + \cdots + 120$ $\Rightarrow d = 4$, $a_1 = 60$

need ↙

$a_n = a_1 + (n-1)d$

$120 = 60 + (n-1)4$

$-60 \quad -60$

$60 = 4(n-1)$

$15 = n-1$

$n = 16$

$S_{16} = \frac{16(60 + 120)}{2}$

$\boxed{S_{16} = 1440}$