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13.1 Sequences

* A <u>sequence</u> is a function whose domain is the set of positive integers.

ex.1) Find the 1st 6 terms of the sequence:

$$\{a_n\} = \left\{\frac{n-1}{n}\right\}$$

$a_1 = \frac{1-1}{1}$, $a_2 = \frac{2-1}{2}$, $a_3 = \frac{3-1}{3}$, $a_4 = \frac{3}{4}$, $a_5 = \frac{4}{5}$, $a_6 = \frac{5}{6}$

$$\boxed{0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots}$$

ex.2) Find 1st 6 terms:

$$\{b_n\} = \left\{(-1)^{n+1}\left(\frac{2}{n}\right)\right\}$$

$b_1 = (-1)^{1+1}\left(\frac{2}{1}\right) = 2$ $b_2 = (-1)^{2+1}\left(\frac{2}{2}\right) = -1$ $b_3 = (-1)^{3+1}\left(\frac{2}{3}\right) = \frac{2}{3}$

$b_4 = -\frac{1}{2}$ $b_5 = \frac{2}{5}$ $b_6 = -\frac{1}{3}$

ex.3) Find 1st 6 terms: $\{c_n\} = \begin{cases} n & \text{if } n \text{ is even} \\ \frac{1}{n} & \text{if } n \text{ is odd} \end{cases}$

$c_1 = \frac{1}{1} = 1$ $c_3 = \frac{1}{3}$

$c_2 = 2$ $c_4 = 4$ $c_5 = \frac{1}{5}$ $c_6 = 6$

$$\boxed{1, 2, \frac{1}{3}, 4, \frac{1}{5}, 6, \ldots}$$

$\longrightarrow$

ex.4) Determine a sequence from a pattern

(a) $e, \frac{e^2}{2}, \frac{e^3}{3}, \frac{e^4}{4}, \ldots \rightarrow a_n = \frac{e^n}{n}$

1st 2nd 3rd 4th

(b) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots \rightarrow b_n = \frac{1}{3^{n-1}}$

1st 2nd 3rd 4th

(c) $1, 3, 5, 7, \ldots \rightarrow c_n = 2n-1$

1st 2nd 3rd 4th

(d) $1, 4, 9, 16, 25, \ldots \rightarrow d_n = n^2$

1 2 3 4 5

(e) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots \rightarrow e_n = (-1)^{n-1}\left(\frac{1}{n}\right)$

1 2 3 4 5

Factorials

* If $n \geq 0$ is an integer, the factorial symbol $n!$ is defined as follows:

$0! = 1 \qquad 1! = 1$

$n! = n(n-1)! \qquad$ if $n \geq 2$

example: $2! = 2 \cdot 1 = 2 \qquad 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$3! = 3 \cdot 2 \cdot 1 = 6$

ex.5) Recursive (given 1st or 1st few terms)

Find 1st 5 terms of:

$s_1 = 1 \quad s_n = n s_{n-1}$

$s_1 = 1$

$s_2 = 2s_{2-1} = 2s_1 = 2(1) = \boxed{2}$

$s_3 = 3s_{3-1} = 3s_2 = 3(2) = \boxed{6}$

$s_4 = 4s_{4-1} = 4s_3 = 4(6) = \boxed{24}$

$s_5 = 5s_{5-1} = 5s_4 = 5(24) = \boxed{120}$

ex. 6) Find 1st 5 terms : $u_1 = 1$ $u_2 = 1$ $u_n = u_{n-2} + u_{n-1}$

$u_1 = 1$

$u_2 = 1$

$u_3 = u_{3-2} + u_{3-1} = u_1 + u_2 = 1 + 1 = \boxed{2}$

$u_4 = u_{4-2} + u_{4-1} = u_2 + u_3 = 1 + 2 = \boxed{3}$

$u_5 = u_{5-2} + u_{5-1} = u_3 + u_4 = 2 + 3 = \boxed{5}$

summation

end → start →

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$$

"the sum of a_k from $k=1$ to $k=n$"

ex. 7) write out each sum :

(a) $\sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

(b) $\sum_{k=1}^{n} k! = 1! + 2! + 3! + \ldots + n!$

ex. 8) write a sum in notation :

(a) $1^2 + 2^2 + 3^2 + \ldots + 9^2 = \sum_{k=1}^{9} k^2$

(b) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^{n-1}} = \sum_{k=1}^{n} \frac{1}{2^{k-1}}$

Properties/Formulas of Sequences:

$$\sum_{k=1}^{n}(ca_k) = ca_1 + ca_2 + \ldots + ca_n = c(a_1 + a_2 + \ldots + a_n) = c\sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n}(a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$$

$$\sum_{k=j+1}^{n} a_k = \sum_{k=1}^{n} a_k - \sum_{k=1}^{j} a_k \quad \text{where } 0 < j < n$$

$$\sum_{k=1}^{n} c = c + c + \ldots + c = cn \quad c \text{ is } \mathbb{R}$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

ex.9) Find the sum of each:

(a) $\sum_{k=1}^{5} 3k = 3\sum_{k=1}^{5} k = 3\left(\frac{5(5+1)}{2}\right) = 3(15) = \boxed{45}$

(b) $\sum_{k=1}^{10}(k^3+1) = \sum_{k=1}^{10} k^3 + \sum_{k=1}^{10} 1 = \left(\frac{10(10+1)}{2}\right)^2 + 1(10)$

$= 3025 + 10 = \boxed{3035}$

(c) $\sum_{k=1}^{24} (k^2-7k+2)$

$\sum_{k=1}^{24} k^2 - 7\sum_{k=1}^{24} k + \sum_{k=1}^{24} 2$

$\frac{24(24+1)(2\cdot 24+1)}{6} - 7\left(\frac{24(24+1)}{2}\right) + 2(24)$

$= 4900 - 2100 + 48 \quad = \boxed{2848}$

(d) $\sum_{k=6}^{20} (4k^2) = 4\sum_{k=6}^{20} k^2 = 4\left[\sum_{k=1}^{20} k^2 - \sum_{k=1}^{5} k^2\right]$

starts at 6

$k=j+1$ so $j=5$

$= 4\left[\frac{20(21)(41)}{6} - \frac{5(6)(11)}{6}\right] = 4[2870-55] = \boxed{11,260}$