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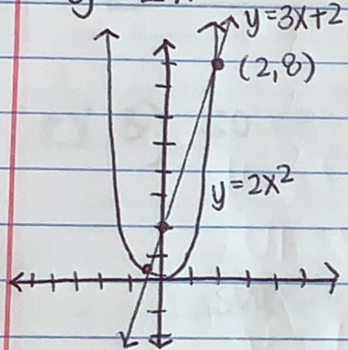
12.6 Systems of Nonlinear Equations

* if system contains two variables & if the equations are easy to graph, then graph them.
* Extraneous solutions can be found when solving nonlinear systems, so you must check solutions!

ex. 1) solve : $\begin{cases} 3x - y = -2 \\ 2x^2 - y = 0 \end{cases}$

if we graph:

$y = 3x + 2$
$y = 2x^2$



(we can see it has 2 solutions where they intersect)

use substitution:

$y = 3x + 2$
$2x^2 - y = 0$
$2x^2 - (3x + 2) = 0$
$2x^2 - 3x - 2 = 0$
$(2x + 1)(x - 2) = 0$
$2x + 1 = 0 \quad x - 2 = 0$
$x = -\frac{1}{2}$ or $x = 2$

use these values to find y:

$y = 3\left(-\frac{1}{2}\right) + 2 = \frac{1}{2}$

or

$y = 3(2) + 2 = 8$

solutions are $x = -\frac{1}{2}, y = \frac{1}{2}$ and $x = 2, y = 8$

check

$x = \frac{1}{2}, y = \frac{1}{2}$

$\begin{cases} 3\left(-\frac{1}{2}\right) - \frac{1}{2} = -\frac{3}{2} - \frac{1}{2} = -2 \\ 2\left(-\frac{1}{2}\right)^2 - \frac{1}{2} = 2\left(\frac{1}{4}\right) - \frac{1}{2} = 0 \end{cases}$

$x = 2, y = 8$

$\begin{cases} 3(2) - 8 = 6 - 8 = -2 \\ 2(2)^2 - 8 = 2(4) - 8 = 0 \end{cases}$

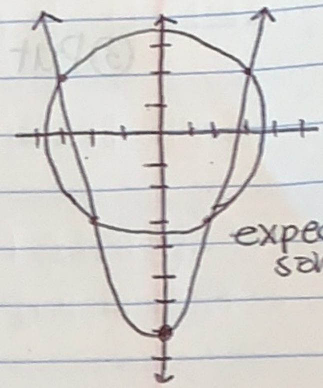
← match original problem ↗

so solutions are $\boxed{\left(-\frac{1}{2}, \frac{1}{2}\right) \text{ and } (2, 8)}$

ex.2) solve: $\begin{cases} x^2+y^2=13 & \leftarrow \text{circle} \\ x^2-y=7 & \leftarrow \text{quadratic} \end{cases}$

radius $=\sqrt{13}=3.6$

$y=x^2-7$



expect 4 solutions

subtract: $\begin{array}{r} x^2+y^2=13 \\ -(x^2-y=7) \\ \hline y^2+y=6 \end{array}$

$y^2-y-6=0$

$(y+3)(y-2)=0$

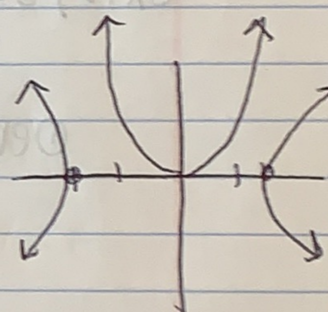
$y=-3 \quad y=2$

if $y=2 \Rightarrow x^2=y+7=9$ so $x=3$ or -3

if $y=-3 \rightarrow x^2=y+7=4$ so $x=2$ or $x=-2$

$\boxed{4 \text{ solutions}: (2,-3),(-2,-3),(3,2),(-3,2)}$

ex.3) solve: $\begin{cases} x^2-y^2=4 & \rightarrow \text{hyperbola} \\ y=x^2 & \rightarrow \text{parabola} \end{cases}$



substitute $y=x^2$ into $x^2-y^2=4$

$y-y^2=4$

so $y^2-y+4=0$ ← no real solutions

$\boxed{\text{inconsistent}}$

ex.4) $\begin{cases} x^2+x+y^2-3y+2=0 \\ \left(x+1+\dfrac{y^2-y}{x}=0\right)x \rightarrow x^2+x+y^2-y=0 \end{cases}$

① eliminate the fractions

② subtract equations: $\begin{array}{r} x^2+x+y^2-3y+2=0 \\ -(x^2+x+y^2-y \quad =0) \\ \hline -2y+2=0 \end{array} \rightarrow y=1$

③ Put $y=1$ into in equation

$$x^2+x+y^2-3y+2=0$$
$$x^2+x+(1)^2-3(1)+2=0$$
$$x^2+x=0$$
$$x(x+1)=0$$
$$x=0 \text{ or } x=-1$$

$x \neq 0$ from domain

$x=-1$ $y=1$ $\boxed{(-1,1)}$

check

$$\begin{cases}(-1)^2+(-1)+1^2-3+2=0 \checkmark \\ -1+1+\frac{1^2-1}{1}=0\end{cases}$$

ex.5) solve: $\begin{cases}3xy-2y^2=-2 \\ 9x^2+4y^2=10\end{cases}$ 2

① eliminate y^2 terms:

$$6xy-4y^2=-4$$
$$9x^2+4y^2=10$$
$$\frac{9x^2+6xy}{3}=\frac{6}{3}$$
$$3x^2+2xy=2$$

② solve for y

$$y=\frac{2-3x^2}{2x} \rightarrow \text{so } x \neq 0$$

③ substitute:

$$9x^2+4y^2=0$$
$$9x^2+4\left(\frac{2-3x^2}{2x}\right)^2=10$$
$$\left(9x^2+\frac{4-12x^2+9x^4}{x^2}=10\right)x^2$$
$$9x^4+4-12x^2+9x^4=10x^2$$
$$\frac{18x^4-22x^2+4=0}{2} \longrightarrow$$

$$9x^4 - 11x^2 + 2 = 0$$

④ Factor: $(9x^2 - 2)(x^2 - 1) = 0$

$$9x^2 - 2 = 0 \qquad x^2 - 1 = 0$$

$$x^2 = \frac{2}{9} \qquad x^2 = 1$$

$$x = \pm\frac{\sqrt{2}}{3} \quad \text{or} \quad x = \pm 1$$

⑤ use $y = \dfrac{2 - 3x^2}{2x}$ to find y:

$$x = \frac{\sqrt{2}}{3} \rightarrow y = \frac{2 - 3\left(\frac{\sqrt{2}}{3}\right)^2}{2\left(\frac{\sqrt{2}}{3}\right)} = \frac{2 - \frac{2}{3}}{2\left(\frac{\sqrt{2}}{3}\right)} = \frac{4}{2\sqrt{2}} = \boxed{\sqrt{2}}$$

$$x = -\frac{\sqrt{2}}{3} \rightarrow y = \frac{2 - \frac{2}{3}}{2\left(-\frac{\sqrt{2}}{3}\right)} = \frac{4}{-2\sqrt{2}} = \boxed{-\sqrt{2}}$$

$$x = 1 \rightarrow y = \frac{2 - 3(1)^2}{2} = \boxed{-\frac{1}{2}}$$

$$x = -1 \rightarrow y = \frac{2 - 3(-1)^2}{-2} = \boxed{\frac{1}{2}}$$

⑥ solutions: $\boxed{\left(\frac{\sqrt{2}}{3}, \sqrt{2}\right), \left(-\frac{\sqrt{2}}{3}, -\sqrt{2}\right), \left(1, -\frac{1}{2}\right), \left(-1, \frac{1}{2}\right)}$