

Name: KEY

PRACTICE Quiz (12.5-12.6)

1. (7 points) Find the partial fraction decomposition of the rational function:

$$f(x) = \frac{x+4}{x^4+4x^2}$$

$$\frac{x+4}{x^2(x^2+4)}$$

$$\frac{x+4}{x^2(x^2+4)} = \left(\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} \right)$$

$$x+4 = Ax(x^2+4) + B(x^2+4) + (Cx+D)x^2$$

$$x+4 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Dx^2$$

$$x^3 \rightarrow 0 = A + C \rightarrow 0 = \frac{1}{4} + C \rightarrow C = -\frac{1}{4}$$

$$x^2 \rightarrow 0 = B + D \rightarrow 0 = 1 + D \rightarrow D = -1$$

$$x \rightarrow 1 = 4A \rightarrow A = \frac{1}{4}$$

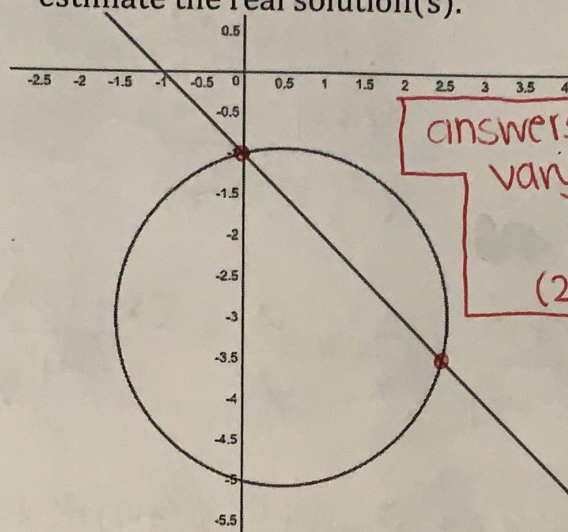
$$C \rightarrow 4 = 4B \rightarrow B = 1$$

$$\frac{x+4}{x^2(x^2+4)} = \frac{1/4}{x} + \frac{1}{x^2} + \frac{-1/4x-1}{x^2+4}$$

3. (7 points) Consider the system of nonlinear equations:

$$\begin{cases} \textcircled{1} & x + y + 1 = 0 \\ \textcircled{2} & x^2 + y^2 + 6y - x = -5 \end{cases}$$

(a) Use the graph of the equations below to estimate the real solution(s).



answers will vary:
 $(0, -1)$
 $(2.5, -3.5)$

(b) Algebraically solve the system of equations given above. Keep solutions as fractions, not decimals. **No points will be awarded if the solutions is found through trial and error.**

$$x + y + 1 = 0$$

$$\rightarrow \text{solve for } y \text{ so } y = -x - 1 \quad \textcircled{3}$$

\rightarrow substitute into equation $\textcircled{2}$

$$x^2 + (-x-1)^2 + 6(-x-1) - x = -5$$

$$x^2 + x^2 + 2x + 1 - 6x - 6 - x = -5$$

$$2x^2 - 5x - 5 = -5$$

$$2x^2 - 5x = 0$$

$$x(2x - 5) = 0 \text{ so } x = 0 \text{ \& } x = \frac{5}{2}$$

\rightarrow plug x's into equation $\textcircled{3}$

$$y = -x - 1$$

$$x = 0 \rightarrow y = -0 - 1 \rightarrow y = -1$$

$$(0, -1)$$

$$x = \frac{5}{2} \rightarrow y = -\frac{5}{2} - 1 = -\frac{5}{2} - \frac{2}{2} = -\frac{7}{2}$$

$$\left(\frac{5}{2}, -\frac{7}{2}\right)$$