

1.6 Equations & Inequalities w/ Absolute Value

HW: #5, 6, 7, 8, 9 - 61 odd, 73, 75

Theorem: $|u| = a$ is equivalent to $u = a$ or $u = -a$

↑ any algebraic expression a positive real #

*if $a = 0$, then $|u| = 0 \equiv u = 0$

*if $a < 0$, then no real solution

Solving equations

ex. 1) $|x+4| = 13$

$x+4 = 13$ or $x+4 = -13$
 $x = 9$ $x = -17$

→ answer $\boxed{\{-17, 9\}}$

ex. 2) $|2x-3| + 2 = 7$ ← isolate ab. value 1st!

$|2x-3| = 5$

$2x-3 = 5$ or $2x-3 = -5$

$2x = 8$

$2x = -2$

$x = 4$

$x = -1$

→ answer $\boxed{\{-1, 4\}}$

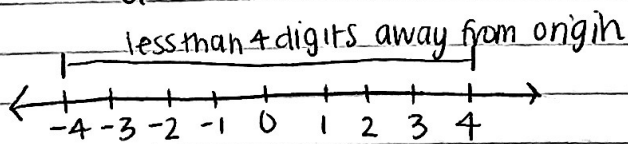
Theorem: $|u| < a$ is equivalent to $-a < u < a$

$|u| \leq a$ is equivalent to $-a \leq u \leq a$

*so $|u| < a \equiv -a < u$ and $u < a$

Solving inequalities

in real terms $\left\{ \begin{array}{l} |x| < 4 \\ \text{so} \\ -4 < x < 4 \end{array} \right.$



ex. 3) $|2x+4| \leq 3$

$-3 \leq 2x+4 \leq 3$

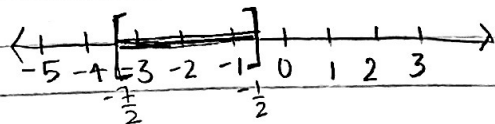
$-4 \quad -4 \quad -4$

$-\frac{7}{2} \leq \frac{2x}{2} \leq \frac{-1}{2}$

$-\frac{7}{2} \leq x \leq -\frac{1}{2}$

$\left\{ x \mid -\frac{7}{2} \leq x \leq -\frac{1}{2} \right\}$

or $\left[-\frac{7}{2}, -\frac{1}{2} \right]$ ← interval notation *



ex. 4) $|1-4x| < 5$

$-5 < 1-4x < 5$

$-1-1 \quad -1$

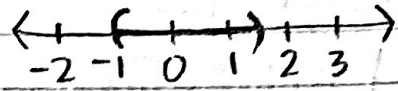
$-6 < -4x < 4$

$-4 \quad -4 \quad -4$

$\frac{3}{2} > x > -1$

$(-1, \frac{3}{2})$ ← interval

rearrange $\{x \mid -1 < x < \frac{3}{2}\}$ ← set

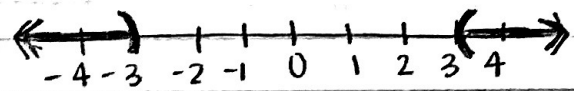


* Theorem: $|u| > a \equiv u < -a \text{ or } u > a$
 $|u| \geq a \equiv u \leq -a \text{ or } u \geq a$

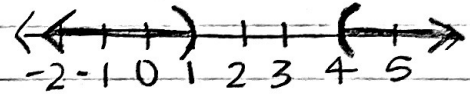
ex. 5) $|x| > 3$ means all x's whose distance is greater than 3 from the origin

$x > 3 \text{ or } x < -3$

$(-\infty, 3) \cup (3, +\infty)$



ex. 6) $|2x-5| > 3$



$2x-5 < -3 \text{ or } 2x-5 > 3$

$2x < 2$

$2x > 8$

$x < 1$

or

$x > 4$

HW: 29. $|x^2-2x| = 3$

$x^2-2x = 3 \text{ or } x^2-2x = -3$

$x^2-2x-3 = 0 \quad x^2-2x+3 = 0$

$(x-3)(x+1) = 0$

↑ cannot factor over \mathbb{R} since $\sqrt{4-4(3)} = \sqrt{-8}$

$x = 3, -1$ so $\{-1, 3\}$

35. $|x^2+3x| = |x^2-2x|$

$x^2+3x = x^2-2x$

$3x = -2x$

$5x = 0$

$x = 0$

$x^2+3x = -(x^2-2x)$

$x^2+3x = -x^2+2x$

$2x^2+1x = 0 \quad x = -\frac{1}{2}$

$x(2x+1) = 0$

$\{-\frac{1}{2}, 0\}$