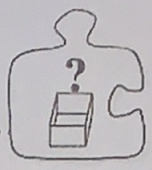
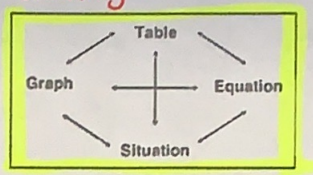
Teacher Notes

1.2.2 How can I model it?



*modeled a geometric relationship by using:



Modeling a Geometric Relationship

Mathematics can be used to model physical relationships to help us understand them better. In this lesson, you will analyze a geometric relationship and look for connections among its multiple representations. You will be given a geometric situation to explore and analyze by gathering and interpreting data. Then you will generalize your findings by creating a mathematical model so that you can make predictions.

Math Notes:

Volume of a Rectangular Prism= length x width x height

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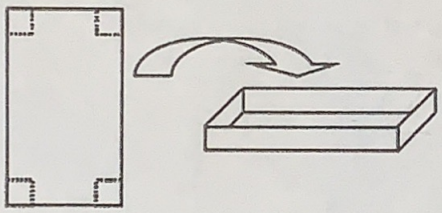
ANALYZING DATA FROM A GEOMETRIC RELATIONSHIP

Your team will make several paper boxes using the instructions given below. Based on the physical models, you will then represent the relationship between the height of each box and its volume in multiple ways.

Cut a sheet of centimeter grid paper to match the dimensions that your team has been assigned. Then, cut the same size square out of each corner and fold the sides up to form a shallow box (with no lid) as shown below.



Dimensions

22 cm × 16 cm	18 cm × 10 cm
22 cm × 14 cm	15 cm × 15 cm
20 cm × 15 cm	15 cm × 10 cm
20 cm × 9 cm	12 cm × 9 cm example

Your Task: As a team you will investigate the relationship between the height of a box (the **input**) and its volume (the **output**). Do not use a graphing calculator today. You may use a scientific calculator.

To analyze this relationship, your team will make at least six different-sized boxes by varying the size of the corners. Begin with six equally-sized pieces of grid paper cut to your assigned dimensions. Record your information using multiple representations—including diagrams, a table, and a complete graph.

Discussion Points

How can we collect data for this relationship?

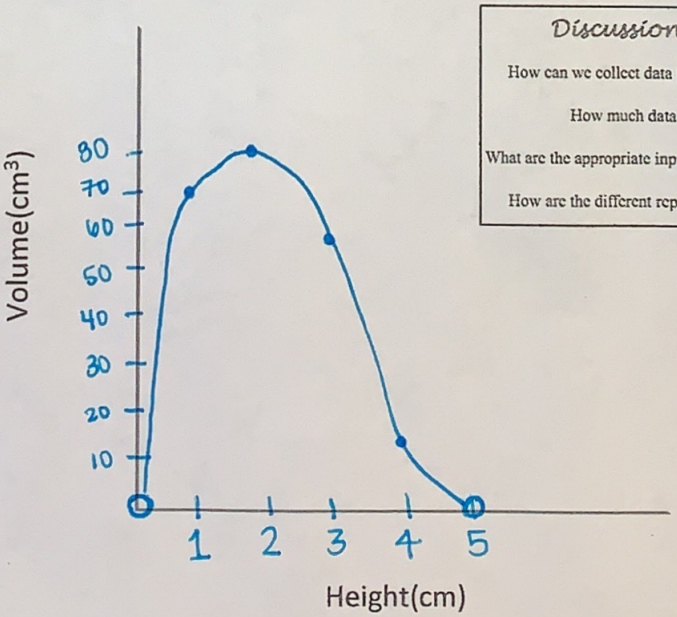
How much data is enough?

What are the appropriate inputs for this relationship?

How are the different representations related?

example:

Height (x)	*Volume* (y)
1	70
2	80
3	54
4	16
⋮	⋮
.	.



Further Guidance

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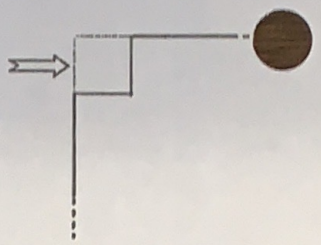
Begin your investigation by building several boxes, taking measurements, and collecting data.



a. As a team, choose a starting input value. Note that this value is the same as *the length of the side of one of the squares cut from the corner of your grid paper*, which becomes the height of your box. Make the first box and determine its volume. Label the box with its important information. Work in the middle of your workspace so that everyone understands what is being measured or calculated, and be sure everyone agrees on the results before recording the information in an input → output table on your own paper.

b. Each team member should now choose a *different* input value and build a new box using this value. Calculate the volumes of the new boxes. Record everyone's data in your table.

c. Use the data in your table to create a complete graph to represent the situation.

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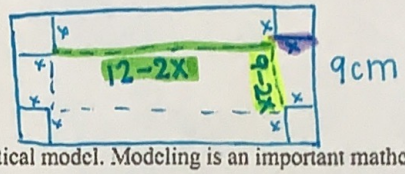
MATHEMATICAL MODELING →



Now you will generalize your results by creating a mathematical model. Modeling is an important mathematical process. A common way to model is to write an equation using algebra.

a. Draw a diagram of one of your boxes. Since the box in this diagram is being used to generalize your results, you want it to represent a relationship between *any* possible input and its output. How can you label the height of the box to represent all possible heights?

Determine the length and width of this box in terms of the height and label your diagram. $l = 12-2x$ $w = 9-2x$ $h = x$

b. Work with your team to write an equation for the volume (output) using the generalized height (input) you chose in part (a). It may help you to remember how you calculated the volume when the height was a number and use the same strategy.

$V = (12-2x)(9-2x)(x)$

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LOOKING FOR CONNECTIONS

Put your table, graph, and equation in the middle of your workspace. With your team, complete the parts below.

As you address each question, justify your statements thoroughly. Also, if you make an observation, discuss how that observation relates your table, graph, and equation.

a. Is the domain of the relationship limited? That is, are there some input values that do not make sense? Why or why not? How can you tell using the graph? The table? The equation? Using the boxes themselves (or diagrams of the boxes)?

b. Is the range of the relationship limited? That is, what are all of the possible volumes (outputs)? Are there any outputs that do not make sense? Why or why not?

c. Should you connect the points on your graph with a smooth curve? That is, should your graph be *continuous* or *discrete*? Explain.

d. Fully describe the graph.