Activity 1.13

Manufacturing Smartphones

Objective

1. Solve a 3×3 linear system of equations.

You recently started your own company to manufacture and sell your latest improvements in the smartphone market. You decided to start with three models: a *basic model* for people who want a smartphone but do not have a lot of disposable income, a *3G model* that has most of the speed and download capabilities that people want, and a *4G model* that has all the latest bells and whistles and is expandable to meet the needs of the immediate future.

You hired and trained your employees to perform all of the basis tasks: the assembly, the testing, and the packaging of each phone. You determined that you have a total of 260 hours for assembly during the week, 170 hours available for testing, and 120 hours available for packaging your phones.

The time allotted for each task on each type of phone is summarized in the following table.

	BASIC MODEL	STANDARD MODEL	DELUXE MODEL
Assembly	1 hour	3 hours	4 hours
Testing	1 hour	2 hours	2 hours
Packaging	1 hour	1 hour	2 hours

Use x, y, and z to represent the number of each type of phone you are to build each week, with x the number of basic models, y the number of 3G models, and z the number of 4G models.

1. Write an equation for the total hours spent on assembling phones each week.

   $x + 3y + 4z = 260$

2. Write an equation for the total hours spent on testing phones each week.

   $x + 2y + 2z = 170$

3. Write an equation for the total hours spent on packaging phones each week.

   $x + y + 2z = 120$

Taken together, the equations in Problems 1–3 form a **3×3 system of linear equations.**

The solution to this system is the ordered triple of numbers (x, y, z) that satisfies all three equations. The strategy for solving such a system is typically to reduce the system to a 2×2 linear system and then proceed to solve this smaller system.

4. Select two equations from Problems 1–3, and use substitution or addition to eliminate one of the variables.

   Using the equations in Problems 1 and 2 to eliminate the variable x, you have

$$\begin{array}{r} x + 3y + 4z = 260 \\ \underline{-x - 2y - 2z = -170} \\ y + 2z = 90 \end{array}$$

5. Select a different pair of equations from Problems 1–3, and eliminate the same variable.

$$\begin{array}{r} x + 3y + 4z = 260 \\ \underline{-x - y - 2z = -120} \\ 2y + 2z = 140 \\ y + z = 70 \end{array}$$

6. The equations from Problems 4 and 5 form a 2×2 system. Solve this new 2×2 system.

$$\begin{array}{r} y + 2z = 90 \\ \underline{-y - z = -70} \\ z = 20 \end{array}$$

   $y + 20 = 70$ or $y = 50$

7. Substitute the solutions from Problem 6 into one of the original three equations. Now solve for the third variable.

Substituting 50 for y and 20 for z into the equation for Problem 3,

$$x + 50 + 2(20) = 120$$
$$x = 30$$

8. How many of each type of phone should you manufacture each week to make optimal use of your available times? Make sure your solution agrees with each of the three original assumptions.

30 basic phones, 50 standard phones, and 20 deluxe phones

9. Explain why it is not possible to solve this 3 × 3 system by graphing on your calculator.

The graphing calculator will only graph functions of two variables, not three.

All of the equations in the 3 × 3 systems in this activity are called *linear equations* even though they cannot all be graphed as single lines. In this case, linearity refers to each variable being linear, that is, raised to the first power.

10. Solve the following 3 × 3 linear system.

$$x - 2y + z = -5 \quad (1)$$
$$2x + y - z = 6 \quad (2)$$
$$3x + 3y - z = 11 \quad (3)$$

If you are not sure where to start, follow these steps.

Step 1. Is it possible to add two of the equations (right side to right side and left side to left side) so that one of the variables is eliminated? [Add equation (1) to equation (2).] Yes.

$$x - 2y + z = -5$$
$$2x + y - z = 6$$
$$3x - y = 1$$

Step 2. Is it possible to add a different pair of equations to eliminate the same variable? [Add equation (1) to equation (3).] Yes.

$$x - 2y + z = -5$$
$$3x + 3y - z = 11$$
$$4x + y = 6$$

Step 3. Notice that your equations from parts a and b form a 2 × 2 linear system. Solve this 2 × 2 system.

Solve $4x + y = 6$ for $y: y = 6 - 4x$.

Substitute $6 - 4x$ for y into $3x - y = 1$.

$$3x - (6 - 4x) = 1$$
$$3x - 6 + 4x = 1$$
$$7x = 7$$
$$x = 1; y = 2$$

Step 4. Substitute your solution from step 3 into any one of the three original equations, and solve the resulting equation for the remaining variable.

$$1 - 4 + z = -5$$
$$z = -2$$
$$\text{so } (x, y, z) = (1, 2, -2)$$

Step 5. The final step is to substitute your potential solution into each of the three original equations. This is the only way you can be confident that your solution is correct.

Check: Equation (1): $1 - 4 - 2 = -5$

Equation (2): $2 + 2 + 2 = 6$

Equation (3): $3 + 6 + 2 = 11$

Most 3 × 3 systems will not have coefficients that are as convenient as the ones you just encountered. The following application provides a case in point.

11. In your job as buyer for Sam's Café, a nationwide coffee bar, you need to buy three grades of coffee bean to be blended with various flavors to make Sam's well-known coffee drinks. This week the three grades of beans are selling for \$0.80, \$1.20, and \$1.80 per pound. Determine the equation that corresponds to each of the following assumptions.

a. The total weight of beans needed is 11,400 pounds.

x = pounds of least expensive; y = pounds of middle grade;

z = pounds of most expensive

$$x + y + z = 11{,}400$$

b. You can only spend \$13,010.

$$0.8x + 1.2y + 1.8z = 13{,}010$$

c. You need 500 more pounds of the least expensive grade than the other two grades combined.

$$x = y + z + 500$$

12. Solve the system you determined in Problem 11. State your solution in terms of the application. Verify that all three assumptions are satisfied.

$$x + y + z = 11{,}400$$
$$8x + 12y + 18z = 130{,}100$$
$$x = y + z + 500$$
$$(y + z + 500) + y + z = 11{,}400$$
$$2y + 2z = 10{,}900$$

$$-8x - 8y - 8z = -91{,}200$$
$$\underline{8x + 12y + 18z = 130{,}100}$$
$$4y + 10z = 38{,}900$$
$$2y + 5z = 19{,}450$$
$$\underline{-2y - 2z = -10{,}900}$$
$$3z = 8550$$

$x = 5950, y = 2600, z = 2850$

You can purchase 5950 pounds of the least expensive grade, 2600 pounds of the middle grade, and 2850 pounds of the most expensive grade.

Appendix

Further examples and practice in solving 3 × 3 linear systems of equations can be found in Appendix A.